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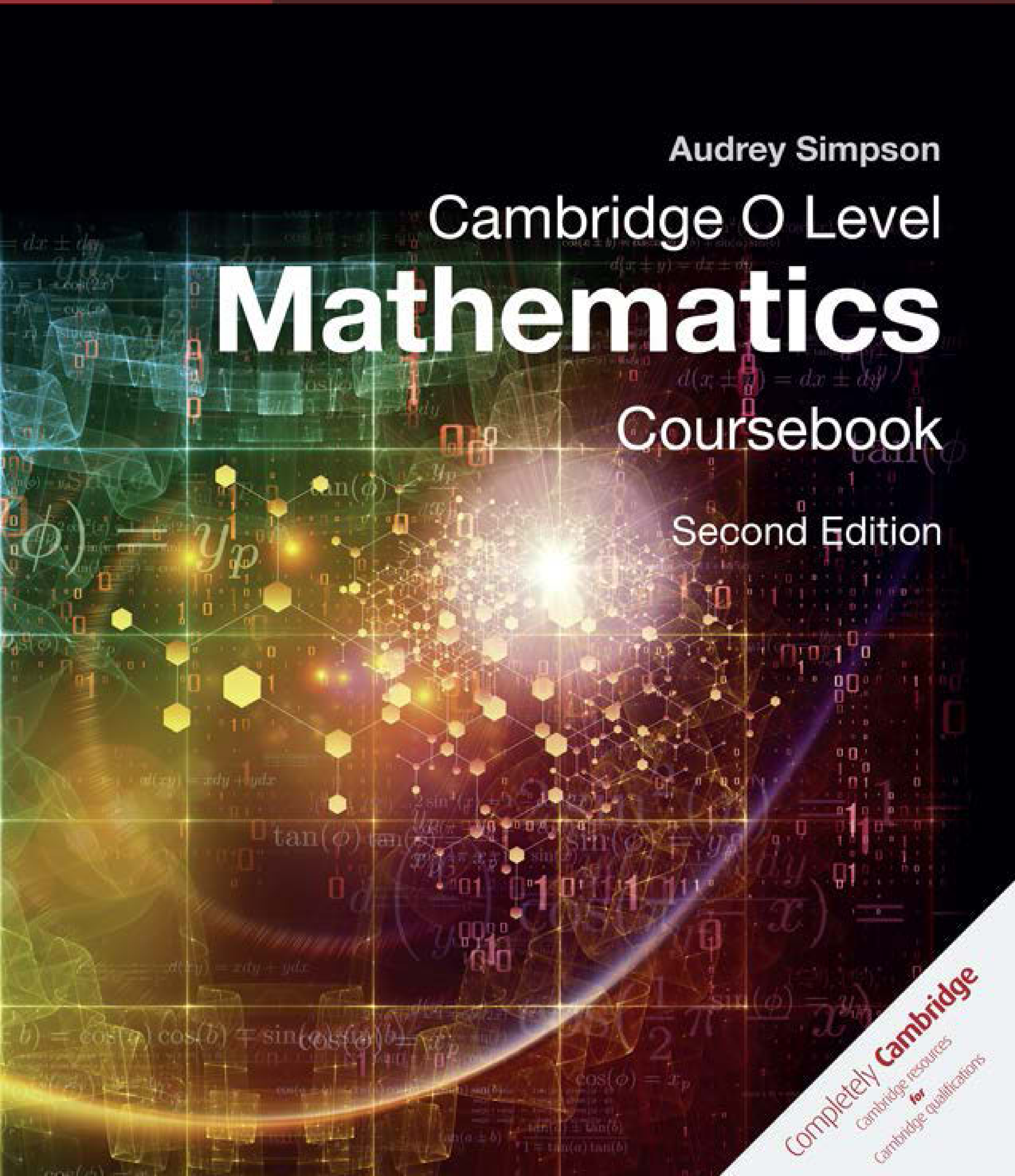
Audrey Simpson

Cambridge O Level

Mathematics

Coursebook

Second Edition



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O Level

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Contents

Introduction

This book covers the entire syllabus for the Cambridge O Level Mathematics from Cambridge International Examinations.

Students will find that the structure of the book allows them to proceed at their own pace through each chapter by:

- working through the essential skills exercise
- reading the explanatory text
- following and understanding the worked examples
- working through each exercise with frequent checking of the answers at the back of the book
- and finally working through the mixed exercise at the end of the chapter.

The book is designed to be worked through sequentially as the required skills and knowledge are built up chapter by chapter and the questions in each chapter only refer to work already covered.

The mixed exercises contain original questions and also carefully chosen questions from past examination papers. These are taken from the O Level examination papers but some appropriate examples are also taken from Cambridge IGCSE papers.

The mixed exercise should consolidate the work covered in the chapter and the past examination questions help students to prepare for examination, and also a sense of achievement that the student has taken steps towards their goal.

The Cambridge O Level Examination consists of two papers. Calculators are not allowed in Paper One, but may be used in Paper Two. This book provides plenty of practice in, and methods for, working without a calculator. Students are encouraged to work without a calculator where possible.

A final section provides suggestions for revision and support as students prepare for examination.

Note to students

- The text in each chapter introduces you to essential mathematical tools.
- The exercises help you gain confidence in using these tools.
- To make the best progress you should ensure that you understand the worked examples. When you have read through each of these examples it can be very helpful to cover up the working and try to reproduce it yourself.
- You should check your answers as you go along. It is important to practise working correctly, and you will not help yourself if you work through a lot of questions

incorrectly before you realise that you have been in error. Of course you will be helping no one if you look up the answer *before* you try the question!

- If you work through the whole of this book you will have covered every topic in the O Level syllabus and will have built up a bank of skills to help you be successful in the future and feel prepared for examination.

Acknowledgements

I would like to thank Professor Gordon Kirby for his invaluable advice and encouragement. I am also grateful for his efforts to check my work patiently for errors, both mathematical and stylistic.

I am also indebted to my sister, Pat Victor, for the times she sorted out frustrating problems with both my computer and the software needed for the production of the manuscript.

Audrey Simpson

Understanding Number 1

Learning Objectives (Syllabus sections 1, 3, 4, 7, 8)

In this chapter you will learn to:

- identify and use different types of number
- express numbers as products of prime numbers
- find the lowest common multiple and highest common factor of two or more numbers
- understand operations and inverses
- recognise common mathematical symbols
- understand and order integers
- convert numbers to and from standard form
- use the recognised order of working in calculations.

1.1 Introduction

By the end of this chapter, you should know more about the different types of numbers that you need to study for the rest of the course. You may feel that you know most of it already, but please work through it as there are plenty of things in it that will help you build the skills you need to be successful in your course. Treat it as revision if you like.

1.2 Essential Skills NO CALCULATOR IN THIS EXERCISE

To get the most from this course, you should know the multiplication tables from 2 to 10 and be able to recall them without hesitation. It is also important to know the facts about addition and subtraction.

Try the following mini-test and see how quickly you can answer the questions without using a calculator.

a 4×6	**b** 3×7	**c** 8×5	**d** 9×8	**e** 2×7
f 6×9	**g** 8×8	**h** 9×5	**i** 7×7	**j** 3×6
k $6 + 7$	**l** $5 + 8$	**m** $9 + 7$	**n** $3 + 5 + 9$	**o** $8 + 9$
p $11 + 9$	**q** $13 + 6$	**r** $3 + 4 + 5$	**s** $16 + 5$	**t** $4 + 17$
u $9 - 4$	**v** $11 - 7$	**w** $15 - 9$	**x** $7 - 4$	**y** $16 \div 8$
z $24 \div 6$				

1.3 Sets of Numbers

Key terms

Natural (or Counting) numbers ($\mathbb{N}$) are the whole numbers you need to count individual items, for example 1, 5, 72, 1000.

Integers ($\mathbb{Z}$) are the counting numbers and also zero and negative whole numbers, for example –50, –2, 0, 11, 251.

The numbers that we use today have developed over a period of time as the need arose. At first, humans needed numbers just to count things, so the simplest set of numbers was the set of **natural** or **counting** numbers. We use the symbol $\mathbb{N}$ to represent the counting numbers, and we use curly brackets to list some of these numbers.

$$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$$

The dots at the end mean 'and so on' because the list goes on forever. (Lists like these are often shown in curly brackets. However, this is not essential.)

When addition and subtraction were introduced, a new set of numbers was needed.

For example, I had three goats. Three were stolen. How many goats do I have now?

We know that the answer is none or zero, which does not appear in the counting numbers.

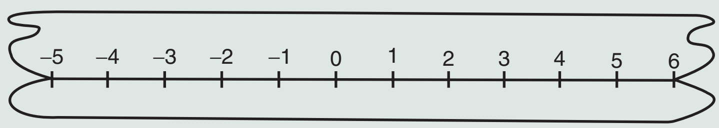
Subtraction also meant that negative numbers were needed, as we will see later in this chapter.

Our next set of numbers is the set of **integers**, which have the symbol $\mathbb{Z}$, and include negative whole numbers, zero and the natural numbers.

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

Practical work

- Make yourself an integer number line on a long strip of paper, like in Figure 1.1.
- Mark on it the integers from –20 through zero to +20. Make sure they are evenly spaced.
- Fold the strip and stick it on the inside cover of your exercise book so that you can unfold it whenever you need it later in the course.



Part of the number line

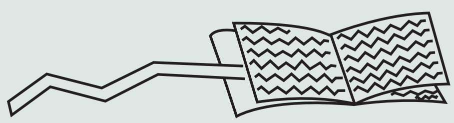


Figure 1.1

Key term

Rational numbers ($\mathbb{Q}$) are the counting numbers, integers and also numbers which can be written as fractions (or ratios), for example $-20, -\frac{3}{4}, 0, 1, 50\frac{1}{2}$.

After addition and subtraction came division and multiplication. What happens when we divide two by three?

The answer is that we get the fraction $\frac{2}{3}$. But where does that fit in with our latest set of numbers? We need another set which includes all the fractions or **rational numbers**. This is the set $\mathbb{Q}$.

Rational numbers can all be expressed as fractions or ratios made up of one integer over another. Remember, for example, that 5 can be written as $\frac{5}{1}$, so integers themselves are included in the set of rational numbers. We can only list some examples of this set because there is an infinite number of members belonging to $\mathbb{Q}$.

Some examples of rational numbers are:

$$\frac{2}{3}, \frac{5}{2}, -2\frac{1}{2}, \frac{3}{100}, 5, 0, 29, -500, \text{ and so on.}$$

Key terms

Real numbers ($\mathbb{R}$) include natural numbers, integers, rational numbers and also irrational numbers.

Irrational numbers are numbers which cannot be written as fractions, for example π, $\sqrt{2}$, $\sqrt{51}$.

The last set we need for our number sets is the set of **real numbers**, $\mathbb{R}$. This includes all the previous sets and also the irrational numbers. **Irrational numbers** are numbers which *cannot* be written as fractions (or ratios) made up of one integer over another.

The Greek letter π (which is spelled and pronounced as pi) is used to represent what is perhaps the most famous irrational number. Pi is the number you get when you divide the length of the circumference of a circle by its diameter. *You can never find the value of π exactly*. We will do some experiments later in the course to see how close we can get to the calculated value of π.

Irrational numbers include square roots of numbers that are not perfect squares themselves, and as we find in the case of π, irrational numbers are decimals that go on and on forever, and never repeat any pattern.

The number π **(= 3.1 4 1 5 9 2 6 5 3 5 8 9 7 9 3 2 3 8 4 6 2 6 4 ...)** has been calculated to billions of places of decimal by high-powered computers, using a more advanced method than measuring the circumference and diameter of a circle. However, no recurring pattern has been found.

Recurring decimals are not irrational numbers because they can always be written as fractions.

For example, $0.666\,666\,666\,66\ldots = \frac{2}{3}$, and $0.285\,714\,285\,714\,285\,714\ldots = \frac{2}{7}$.

Recurring decimals do, of course, have a repeating pattern, unlike irrational numbers.

Write down the sequence of numbers that recur in the decimal equivalent of $\frac{2}{7}$.

Figure 1.2 will help you to see how these sets of numbers build up.

Each number type has been drawn with two or three examples in it.

Another way to show these sets is on number lines like in Figure 1.3. Some examples of each set are shown below. The arrows show that the sets go on forever in that direction.

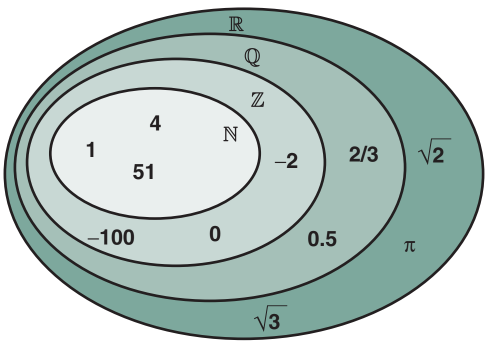


Figure 1.2 Number sets

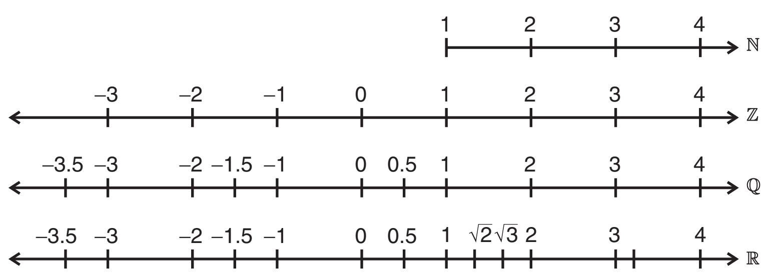


Figure 1.3 Number lines

Example 1

2	$\sqrt{3}$	$\frac{1}{1000}$	−99	$2\frac{1}{2}$	$-\frac{1}{4}$	π	0.3	0	2005

From the list given above, select:

a the natural numbers **b** the integers **c** the rational numbers

d the irrational numbers **e** the real numbers.

Answer 1

a The natural numbers ($\mathbb{N}$) are: 2 and 2005.

b The integers ($\mathbb{Z}$) are: –99, 0, 2 and 2005 (because each larger set includes the set before it).

c The rational numbers ($\mathbb{Q}$) are: $-99, -\frac{1}{4}, 0, \frac{1}{1000}, 0.3, 2\frac{1}{2}, 2, 2005$.

d The irrational numbers are: $\sqrt{3}$ and π (because these are decimals that go on forever with no repeating pattern).

e The real numbers ($\mathbb{R}$) are: $2, \sqrt{3}, \frac{1}{1000}, -99, 2\frac{1}{2}, -\frac{1}{4}, \pi, 0.3, 0, 2005$.

Key terms

Prime numbers are divisible only by themselves and 1 without leaving a remainder, for example 2, 11, 37, 101.

Factors of a number can be multiplied together to make that number, for example 1, 2, 3 and 6 are factors of 6.

Multiples of a number are the result of multiplying that number by any of the natural numbers, for example 6, 12, 36 and 600 are multiples of 6.

A number which can be divided by another number without leaving a remainder is said to be **divisible** by that number. For example, 39 is divisible by 3.

Within the above sets of numbers there are other, smaller sets. Some of these sets are discussed below.

1.4 Prime Numbers, Factors and Multiples

In this section we will use natural numbers only.

Prime numbers are natural numbers that are only **divisible** by themselves or by 1.

Some examples of prime numbers are:

$$2, 3, 5, 7, 11, 13, 17, \ldots$$

Notice that 1 is *not* counted as a prime number, and 2 is the only *even* prime number.

Example 2

Write a list of all the prime numbers between 20 and 35.

Answer 2

23, 29, 31

(All the other numbers between 20 and 35 are divisible by numbers other than just themselves or 1.)

The **factors** of a number are the natural numbers that can be multiplied together to make the number.

For example, 2 and 3 are factors of 6 because $2 \times 3 = 6$.

The **multiples** of a number are obtained by multiplying the number by other natural numbers.

For example, the multiples of 12 would include 12, 24, 36, 48 and so on.

The factors of 12 in Figure 1.4 are shown multiplied together. This is called a **product of factors**. So numbers that are multiplied together are called **factors**, and the result of multiplying them together is called the **product**. There are other factors of 12.

NOTE:

You may need to find a way of remembering which are factors of a number, and which are multiples of the number.

Perhaps you can remember that multiples are bigger than the original number, or that they are in the multiplication tables (times tables) for that number.

So the multiples of 2 are 2, 4, 6, 8, 10, …

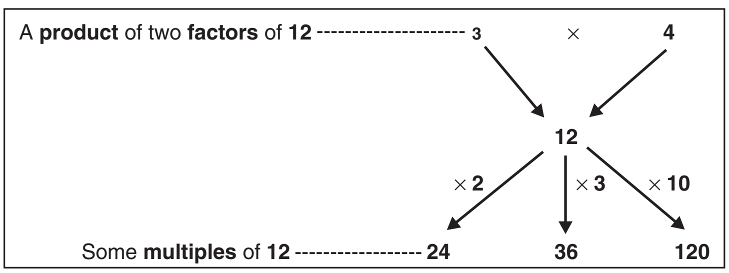


Figure 1.4 Factors and multiples of 12

Altogether the factors of 12 are: 1, 2, 3, 4, 6 and 12 (all the numbers that will divide into 12 without leaving a remainder).

Of particular interest are the prime factors. The prime numbers among the factors of 12 are 2 and 3. We can write 12 as a **product** of its **prime factors**:

$$12 = 2 \times 2 \times 3$$

or we can **list** the prime factors of 12: {2, 3}.

A factor tree is a neat method for finding prime factors of larger numbers. The following example will show you how to make a factor tree.

Example 3

Write 200 as a product of its prime factors.

Answer 3

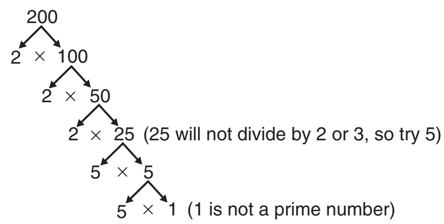
First make a list of the smaller prime numbers: 2, 3, 5, 7, …

Start by dividing by 2, and repeat until the number will no longer divide by 2.

Then work through your list in order, trying 3, then 5 and so on.

The answer is: $200 = 2 \times 2 \times 2 \times 5 \times 5$.

(Check this by multiplying out.)



Example 4

a List all the factors of 18.

b List the prime factors of 18.

c Write 18 as a product of its prime factors.

d List three multiples of 18.

Answer 4

a {1, 2, 3, 6, 9, 18}

b {2, 3}

c $18 = 2 \times 3 \times 3$

d For example, 36 (18×2), 54 (18×3), 90 (18×5).

Exercise 1.1

NO CALCULATOR IN THIS EXERCISE

1 5, −100, −3.67, π, 0, 1507, $\frac{99}{7}$, $\frac{6}{1}$

From the list above:

a Write down all the real numbers.

b Write down all the rational numbers.

c Write down all the integers.

d Write down all the natural numbers.

e One of the numbers is irrational. Which is it?

2 **a** List all the factors of 30.

b List the prime factors of 30.

c Write 30 as a product of its prime factors. (Multiply out to check your answer.)

d Write down three multiples of 30.

3 1, 4, 30, 45, 5, 15, 9, 1500, 3, 10

From this list choose:

a the multiples of 15

b the factors of 15.

4 Use a factor tree to find the prime factors of 240. Write your answer:

a as a list of prime factors

b as a product of prime factors.

5 Write down all the prime numbers between 20 and 40.

6 Which of the following numbers are prime numbers?

37, 49, 53, 81, 87, 93, 101

7 Write down a list of numbers between 80 and 90, including 80 and 90.
From your list find:
a two prime numbers **b** three multiples of 5 **c** a factor of 348.

1.5 Highest Common Factor (HCF) and Lowest Common Multiple (LCM)

Key terms

The Highest Common Factor (HCF) of two or more given numbers is the highest number which will divide into both or all of the given numbers without leaving a remainder, for example 3 is the HCF of 9, 12 and 15.

The Lowest Common Multiple (LCM) of two or more given numbers is the lowest number which is a multiple of both or all of the given numbers, for example 18 is the LCM of 2, 6 and 9.

'Common' in this case means 'belonging to all'.

We often need to find the factors or multiples of two (or more) numbers that belong to both (or all) the numbers. One way to do this is to list all the factors or multiples of both numbers and see which factors or multiples occur in both lists.

The following example shows how this is done.

Example 5

a **i** List all the factors of 30. **ii** List all the factors of 20.
iii From your two lists find the common factors of 20 and 30 (not including 1).

b **i** List the first four multiples of 30 (not including 30 itself).
ii List the first five multiples of 20 (not including 20 itself).
iii From your two lists, find any common multiples.

c Find the HCF of 30 and 20. **d** Find the LCM of 30 and 20.

Answer 5

a **i** {1, 2, 3, 5, 6, 10, 15, 30} **ii** {1, 2, 4, 5, 10, 20} **iii** {2, 5, 10}
b **i** {60, 90, 120, 150} **ii** {40, 60, 80, 100, 120} **iii** {60, 120}
c 10 **d** 60

Using the above example you should see that finding the highest common factor (HCF) of 20 and 30 is simple. It is the highest number that appears in both lists of factors of both the numbers. The HCF of 20 and 30 is 10.

Similarly, the lowest common multiple of 20 and 30 is the smallest number that appears in both lists of multiples. The LCM of 20 and 30 is 60.

An alternative method for finding the HCF of two or more numbers is first to write them as products of their prime factors, and then pick out the factors common to both lists. The example shows this.

Example 6

a Write
i 360 and **ii** 980
as products of their prime factors.

b Find the highest common factor of 360 and 980

Answer 6

a **i** $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$ **ii** $980 = 2 \times 2 \times 5 \times 7 \times 7$
b $\text{HCF} = 2 \times 2 \times 5 = 20$

1.6 Tests of Divisibility without Using a Calculator

Before you go any further you might like to try some tests of divisibility which can help you save time in these questions. These tests show what will divide into a number without leaving a remainder.

- **Divisibility by 2:** All even numbers divide by 2. (All even numbers end in 2, 4, 6, 8 or 0.)
- **Divisibility by 3:** This is a rather surprising test, but it does work!

Add all the digits (individual numbers) of the entire number together. If the result is 3, 6 or 9 then the number will divide by 3. If the result is 10 or more, keep adding the digits until you get to a single digit. This is called finding the **digital root** of the number. If the digital root is 3, 6 or 9 then the number will divide by 3.

For example, the digital root of 2115 is $2 + 1 + 1 + 5 = 9$, so 2115 will divide by 3 (check it on your calculator).

Of course, it does not matter what order the digits of the number appear or if any zeroes appear in the number, so 5121, 2511, 12 510, 105 120 (and so on) will all divide by 3.

To find the digital root of 3672:

$$3 + 6 + 7 + 2 = 18$$
$$1 + 8 = 9$$

So the digital root of 3672 is 9. Hence, 3672 will divide by 3.

- **Divisibility by 5:** All numbers ending in 5 or 0 will divide by 5. Therefore, 3672 will not divide by 5 whereas 3670 will.
- **Divisibility by 6:** All *even* numbers with a digital root of 3, 6 or 9 will divide by 6. 3672 will divide by 6.
- **Divisibility by 9:** All numbers with a digital root of 9 will divide by 9. 3672 will divide by 9.

Example 7

a Test 552 for divisibility by 2, 3, 5, 6 and 9.

b Test 6165 for divisibility by 2, 3, 5, 6 and 9.

Answer 7

a 552 is even, so it will divide by 2.
$5 + 5 + 2 = 12 \rightarrow 1 + 2 = 3$, so it will divide by 3.
552 does not end in 5 or 0, so it will not divide by 5.
552 is even *and* it will divide by 3, so it will also divide by 6.
The digital root of 552 is 3, not 9, so it will not divide by 9.

b 6165 is not even, so it will not divide by 2.
$6 + 1 + 6 + 5 = 18 \rightarrow 1 + 8 = 9$, so it will divide by 3.
6165 ends in 5, so it will divide by 5.
Although 6165 will divide by 3 it is not even, so it will not divide by 6.
The digital root of 6165 is 9, so it will divide by 9.

Exercise 1.2

NO CALCULATOR IN THIS EXERCISE

1 **a** List all the factors of 8. Then list all the factors of 12.
b Find the highest common factor of 8 and 12.

2 Find the highest common factor of 21 and 42.

3 **a** List all the factors of:
i 15 **ii** 35 **iii** 20
b Write down the highest common factor of 15, 35 and 20.

4 **a** List the first six multiples of 12 and of 8.
b Write down the lowest common multiple of 12 and 8.

5 Find the lowest common multiple of 3, 5 and 12.

6 Test 21603 for divisibility by 2, 3, 5 and 9. Explain your reasoning (see Example 6).

7 Test 515196 for divisibility by 2, 3, 5, 6 and 9. Explain your reasoning.

1.7 Operations and Inverses

Mathematical operations like addition, or division, have inverses which 'undo' the operation.

For example, $2 \times 3 = 6$, and $6 \div 3 = 2$.

Division is the inverse of multiplication because it 'undoes' multiplication.

Also, multiplication is the inverse of division, as you can see in Figure 1.5.

What do you think is the inverse of addition? Look at Figure 1.6.

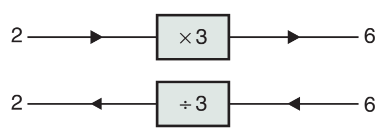


Figure 1.5 Multiplication and division are inverses

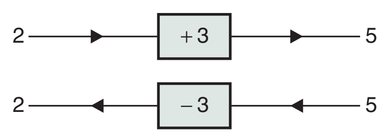


Figure 1.6 Addition and subtraction are inverses

1.8 Squares and Square Roots, Cubes and Cube Roots

The square of a number is the result of multiplying a number by itself.

For example, the square of 9 is $9 \times 9 = 81$, the square of 11 is $11 \times 11 = 121$, and the square of 35 is $35 \times 35 = 1225$.

The compact way of showing that the number is to be squared is to write it to the power of 2.

So the square of 9 is written as $9^2 = 81$. Similarly, $11^2 = 121$ and $35^2 = 1225$.

Finding the square root of a number *undoes* the squaring, so for example, the square root of 81 is 9, the square root of 121 is 11, and the square root of 1225 is 35.

The compact way of showing that the square root of a number is to be found is to use the square root sign: $\sqrt{\ }$.

So the square root of 81 is written as $\sqrt{81} = 9$; also, $\sqrt{121} = 11$ and $\sqrt{1225} = 35$.

Key term

An **inverse operation** reverses the effect of another operation. For example, divide and multiply are inverses of each other, or square and square root.

You should be able to see that squaring and finding the square root undo each other.

As we have seen above, operations which 'undo' each other are inverses of each other.

Hence, squaring and finding the square root are **inverse operations** (see Figure 1.7).

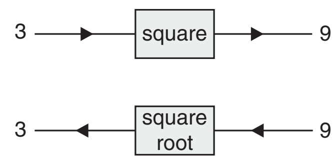


Figure 1.7 Square and square root are inverses

We will come across more inverse operations later in the course.

We can find the square of any number. My calculator tells me that the square of 2.41 is 5.8081. It also tells me that $\sqrt{468.2896} = 21.64$.

However, not all numbers have exact square roots. For example, $\sqrt{2}$, $\sqrt{3}$ or $\sqrt{5}$ are numbers with decimals that 'go on forever' without any repeating pattern; they are irrational numbers.

NOTE:

It would be helpful to learn to recognise some of the irrational numbers, such as π, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$ and $\sqrt{10}$, so that you can give examples when required.

$\sqrt{49} = 7$ exactly, so $\sqrt{49}$ is a rational number. As we see above, $\sqrt{468.2896} = 21.64$, so $\sqrt{468.2896}$ is a rational number.

My calculator tells me that $\sqrt{7.2} = 2.683\,281\,573$ before it runs out of space on its display, so $\sqrt{7.2}$ *looks* as if it could be an irrational number, although we cannot tell for certain by this method alone.

The numbers that you get when you square the natural numbers are called **perfect squares**.

They are called perfect squares because on being square rooted they give whole numbers.

The first four perfect squares are 1, 4, 9, 16. Write down the next three square numbers.

NOTE:

It would be useful to make a list of the first five square numbers and the first five cube numbers and learn to recognise them. You will come across them quite often.

The **cube** of a number is the result of multiplying that number by itself twice.

The cube of $4 = 4 \times 4 \times 4 = 16 \times 4 = 64$.

The compact way of writing the cube of a number is to write it to the power 3, so the cube of 4 is: $4^3 = 4 \times 4 \times 4 = 64$.

The cubes of the first four numbers are 1, 8, 27, 64. What is the cube of the next number?

Cube numbers are also often called **cubic numbers**.

The inverse of cubing a number is finding the cube root, and you may find that your calculator has a cube root button if you look carefully.

The cube root sign is $\sqrt[3]{\ }$, so $\sqrt[3]{27} = 3$ (because $3 \times 3 \times 3 = 27$).

Example 8

a 12, 6, 7, 36, 125, 5, 15, 4

From the list of numbers choose:

i a perfect square **ii** the square root of 49 **iii** 6^2
iv $\sqrt{25}$ **v** 5^3

b **Use your calculator** to find:

i 42.3^2 **ii** $\sqrt{9.61}$ **iii** 1.6^3 **iv** $\sqrt[3]{64}$

Answer 8

a **i** 36 or 4 **ii** 7 **iii** 36
iv 5 **v** 125

b **i** 1789.29 **ii** 3.1
iii 4.096 **iv** 4

Exercise 1.3

NO CALCULATOR UNLESS SPECIFIED

1 For each of the operations below state the inverse.
a multiply **b** subtract **c** square **d** cube root

2 Write down:
a the square of 6 **b** the square root of 9
c 2^3 **d** $\sqrt{25}$ **e** 10^2 **f** 10^3

3 **Use your calculator** to find:
a 5.2^2 **b** $\sqrt{82.81}$ **c** $\sqrt{100}$ **d** $\sqrt[3]{1000}$

4 $\sqrt{256}$ $\sqrt{6.1}$ $\sqrt{841}$ $\sqrt{7}$ $\sqrt{449.44}$
Use your calculator to choose from the above list:
a three numbers that you think are rational
b two numbers that you think are irrational.

In each case write down all the figures on your calculator display.

5 Write a list of the first seven square numbers.

6 Fill in the gaps in this list of cube numbers.

1, 8, …, 64, …, 216.

7 Using your answers to questions 5 and 6, write down a number which is both a perfect square and a perfect cube.

8 **Using your calculator**, find another number which is both a perfect square and a perfect cube.

9 1 2 3 4 5 6 7 8 9 10 11

Copy Table 1.1. Enter each of the numbers in the list above in the correct rows in your table. (Some numbers may fit in more than one row.)

Natural numbers	
Prime numbers	
Even numbers	
Multiples of 3	
Square numbers	
Cube numbers	
Factors of 20	

Table 1.1 Number types

1.9 Directed Numbers

Key term

Directed numbers are numbers that can be positive as well as negative. The sign indicates a direction, for example −10 °C is 10 °C *below* freezing.

We have looked at integers, which are positive (with a plus sign) or negative (with a minus sign) whole numbers, or zero, which has no sign.

Directed numbers are also positive or negative but include the whole set of real numbers, hence they also include rational and irrational numbers, as well as integers.

They are called **directed numbers** because they indicate a direction along a number line.

Think of a thermometer that measures temperatures above and below zero.

If the temperature starts at 4 °C and *falls* by 5 °C, it will end at –1 °C. This can be written as 4 – 5 = –1.

The minus sign in front of the 5 shows the direction in which the temperature has moved from 4.

The minus sign in front of the 1 shows that it is 1 degree *below* zero. If the temperature starts at 4 °C and *rises* by 5 °C, it will end at +9 °C. This can be written as 4 + 5 = +9.

The plus sign shows that the temperature is 9 degrees *above* zero. In practice we do not usually write in the plus sign. If a number is written without a sign it is assumed that it is positive. We are not restricted to whole numbers, so 4 – 5.5 = –1.5.

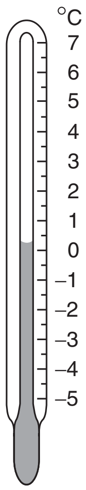


Figure 1.8

Example 9

a Use the thermometer shown in Figure 1.8 to find the new temperature in each case below.

- **i** The temperature starts at –5 °C and rises by 4 °C.
- **ii** The temperature starts at –1 °C and falls by 2 °C.
- **iii** The temperature starts at –2.5 °C and rises by 5.5 °C.

b Use the thermometer to work out the following:

- **i** 3 – 6
- **ii** –5 + 9
- **iii** –1 – 3.5
- **iv** 3 – 5 + 6
- **v** the difference between 4 °C and 7 °C
- **vi** the difference between –2 °C and –4 °C
- **vii** the difference between –2 °C and 4 °C.

c Which is warmer, 2 °C or –5 °C?

Answer 9

a

- **i** –5 + 4 = –1, so the new temperature is –1 °C.
- **ii** –1 – 2 = –3, so the new temperature is –3 °C.
- **iii** –2.5 + 5.5 = +3, so the new temperature is +3 °C, (or just 3 °C).

b

- **i** 3 – 6 = –3
- **ii** –5 + 9 = + 4 (or just 4)
- **iii** –1 – 3.5 = –4.5
- **iv** 3 – 5 + 6 = – 2 + 6 = 4
- **v** 3 °C (look at the thermometer)
- **vi** 2 °C
- **vii** 6 °C

c 2 °C is warmer than –5 °C.

Exercise 1.4

NO CALCULATOR IN THIS EXERCISE

1 Draw a thermometer, with temperatures between –10 °C and +10 °C.
Use it to complete the following statements.

- **a** –10 + 5 =
- **b** –2 – 3 =
- **c** 5 – 8 =
- **d** 0 – 7 =
- **e** 6 + 2 – 3 =

2 Figure 1.9 shows a marker in a reservoir which is used to show the level (in metres) of the water. Copy the diagram and use it to answer the following questions.

- **a** Overnight the water level sinks from the level shown in the diagram to –1.5 metres. By how many metres has the water level in the reservoir fallen?
- **b** The water level falls another 2.1 metres. What is the new level?
- **c** By how much does the water have to rise to bring the level up to 2 metres?

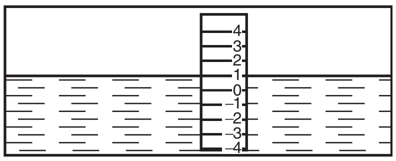


Figure 1.9 Water level

3 Figure 1.10 shows the cross-section of a mountain region. Sea level is 0 metres. A climber starts at 15 metres *below* sea level and climbs 100 metres. How high is he above sea level now?

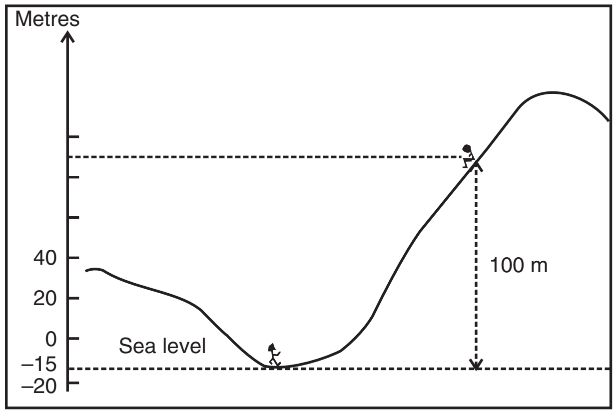


Figure 1.10

4

Bank Account			
Start	Money in	Money out	Balance
–$216			–$216
	$503		(a)
		$290	(b)
	(c)		$0.00

Table 1.2 Bank statement

My bank account is overdrawn by $216. The balance (the amount of money I have in the bank) is shown in the first line in Table 1.2 as –$216. This means that I owe the bank $216.

a I pay in $503. What should my account balance show now?
b I write a cheque for $290 to pay for my electricity. Am I still overdrawn?
c If so, how much would I need to pay in to clear my debt?

We will learn more about directed numbers in Chapter 3.

1.10 Important Mathematical Symbols

You are already familiar with some mathematical symbols.

For example, $+$, $-$, $\times$, $\div$, π, $\sqrt{\ }$ and $=$.

Another symbol which is sometimes used is $\neq$, which means 'is not equal to'.

For example, $4 \neq 7$, or 'four is not equal to seven'.

We also need to be able to use symbols to mean 'is greater (or larger) than', or 'is less (or smaller) than'.

For example, we need a mathematical way of writing 'four is less (or smaller) than seven'.

This is written as $4 < 7$.

NOTE:
If you have difficulty remembering the inequality signs, you might be able to remember that the inequality sign points to the smaller number, or even that the smaller end of the sign is on the side of the smaller number.

We can also write $7 > 4$. This means that 'seven is greater than four'.

Suppose we wanted to say that the number of days in February is greater than or equal to 28? This would be written as: Number of days in February $\geqslant 28$.

So $\geqslant$ means greater than *or equal to* and $>$ means *strictly* greater than. What do you think $\leqslant$ means?

The signs $>$ and $<$ are called **inequality signs**.

1.11 Ordering Integers

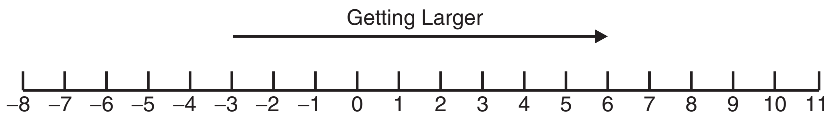


Figure 1.11 The number line

The number line in Figure 1.11 shows the integers from –8 to 11. The rest of the real numbers fit in their correct places along the line, so –2.5 would be halfway between –3 and –2.

The numbers get ***larger*** as you go from ***left*** to ***right***.

For example, $8 > 3$ (as we know).

Also, $1 > -2$, $-4 < 0$ and so on.

This is also true for all the positive and negative real numbers, so $-6.25 < 3.5$.

Example 10

Use the number line you made earlier to insert the correct symbol between the following pairs of numbers:

a 7 … 20 **b** –5 … 10 **c** 2 … –1

d –8 … –19 **e** 4.5 … –6.5 **f** $-\frac{1}{2} \ldots -4$

Answer 10

a $7 < 20$ **b** $-5 < 10$ **c** $2 > -1$

d $-8 > -19$ **e** $4.5 > -6.5$ **f** $-\frac{1}{2} > -4$

Exercise 1.5

NO CALCULATOR IN THIS EXERCISE

1 Write down the symbol for:

a pi **b** square root **c** cube root

d is not equal to **e** is less than **f** is greater than or equal to

2 Fill in the correct inequality sign between each of the following pairs of numbers.

a 2 … 4 **b** –2 … –5 **c** –10 … 4 **d** –1 … 0

3 Arrange the following integers in the correct order, starting with the smallest.

100, –1, –100, 0, –89, –76, 75, 101, 61, –62

1.12 Standard Form

Sometimes we have to work with very large numbers (the distance from earth to the moon is approximately 384 400 000 metres), or very small numbers (the thickness of a page in one of my books is approximately 0.000 021 3 metres).

There is a neater way of writing these numbers without having to use so many zeroes.

It is called **standard form**. Using standard form we write numbers in the form $a \times 10^n$, where a is a number greater than or equal to 1 and less than 10 ($1 \leqslant a < 10$) and n is an integer.

To write 230 000 (two hundred and thirty thousand) in standard form:

- First identify the place where the decimal point belongs (since it is not shown). We know that if the decimal point is not shown, it actually comes after the last digit.
  So 230 000 could be written as 230 000.0.
- Next count how many places the decimal point would have to be moved back until it is between the 2 and the 3. You will see that it is 5 places.
- So $230\,000 = 2.3 \times 10^5$. This is read as 'two hundred and thirty thousand is equal to two point three times ten to the power five'.

You will learn more about powers and working with standard form in a later chapter. For the moment, you just have to understand how to write numbers in standard form.

To write 0.000 003 546 in standard form:

- Count how many places the decimal point would have to be moved forward to lie between 3 and 5. It is 6 places.
- So $0.000\,003\,546 = 3.546 \times 10^{-6}$ (this is ten to the power of **negative** six).

NOTE:
If it is a problem to remember which power to use, you should notice that numbers less than one have a negative power and numbers greater than 10 have a positive power in standard form.

Example 11

a Write in standard form:

i 20 015 **ii** 175 **iii** 3 200 000
iv 0.127 **v** 0.005 06

b Write in the normal way:

i 9.013×10^{-3} **ii** 1.0007×10^7

Answer 11

a **i** 2.0015×10^4 **ii** 1.75×10^2 **iii** 3.2×10^6
iv 1.27×10^{-1} **v** 5.06×10^{-3}

b **i** 0.009 013 **ii** 10 007 000

1.13 Order of Working in Calculations

Ram was asked to calculate $5 + 2 \times 3$, without using a calculator. His answer was 21.

He checked his answer with a calculator. The calculator answer was 11.

What has happened?

Both Ram and the calculator were correct in different ways.

Ram first added 5 and 2 and then multiplied by 3 ($5 + 2 = 7$ then $7 \times 3 = 21$).

The calculator multiplied 2 and 3 first and then added 5 ($2 \times 3 = 6$ then $6 + 5 = 11$).

It is clearly not satisfactory to get two different answers to the same question, so an order of working had to be decided to ensure that all calculations yield the same answer.

NOTE:
There are different ways of remembering this order. For example, the made-up word BoDMAS is often used. You could say that the 'o' stands for 'of', which usually means multiply, as in $\frac{1}{7}$ of $35 = \frac{1}{7} \times 35$.

The accepted order is:

- First **B**rackets.
- Next **D**ivision and **M**ultiplication (in either order).
- Lastly **A**ddition and **S**ubtraction (in either order).

Try to follow this example.

To calculate $7 + 3 \times 2 - (6 - 2) \div 2$,

B (brackets)	$= 7 + 3 \times 2 - \mathbf{4} \div 2$	$[(6 - 2) = \mathbf{4}]$
D (division)	$= 7 + 3 \times 2 - \mathbf{2}$	$[4 \div 2 = \mathbf{2}]$
M (multiplication)	$= 7 + \mathbf{6} - 2$	$[3 \times 2 = \mathbf{6}]$
A (addition)	$= \mathbf{13} - 2$	$[7 + 6 = \mathbf{13}]$
S (subtraction)	$= \mathbf{11}$	$[13 - 2 = \mathbf{11}]$

Answer: $7 + 3 \times 2 - (6 - 2) \div 2 = 11$

Try putting this in your calculator in exactly the same order as it is written and see if your calculator arrives at the same answer when you press the 'equals' button. Most calculators now use this form of logic (order of working), but you need to be sure about your own.

There will be more about this in Chapter 4.

Work out $4 \times 6 \div 2$ by doing the multiplication first.

$$4 \times 6 \div 2 = 24 \div 2 = 12$$

Now do the same sum but do the division first.

$$4 \times 6 \div 2 = 4 \times 3 = 12$$

You should note that multiplication and division can be done in either order. Can you find a rule for addition and subtraction?

It is very important that you learn this order of working, and know how to use it.

Example 12

Work out the following, showing your working:

a $4 + 3 \times 10 - 6 \div 2$ **b** $(4 + 3) \times 10 - 6 \div 2$ **c** $4 + (3 \times 10) - 6 \div 2$

d $4 + 3 \times (10 - 6) \div 2$ **e** $(4 + 3) \times (10 - 6) \div 2$ **f** $4 + (3 \times 10 - 6) \div 2$

Answer 12

a $4 + 3 \times 10 - 6 \div 2$
$= 4 + 30 - 3 = 31$

b $(4 + 3) \times 10 - 6 \div 2$
$= 7 \times 10 - 3$
$= 70 - 3 = 67$

c $4 + (3 \times 10) - 6 \div 2$
$= 4 + 30 - 3 = 31$

(This is the same as **(a)** because the multiplication is done first anyway, and so does not need brackets.)

d $4 + 3 \times (10 - 6) \div 2$
$= 4 + 3 \times 4 \div 2$
$= 4 + 6 = 10$

(Notice that $3 \times 4 \div 2 = 12 \div 2 = 6$ *or* $3 \times 4 \div 2 = 3 \times 2 = 6$)

e $(4 + 3) \times (10 - 6) \div 2$
$= 7 \times 4 \div 2 = 14$

f $4 + (3 \times 10 - 6) \div 2$
$= 4 + (30 - 6) \div 2$
$= 4 + 24 \div 2$
$= 4 + 12$
$= 16$

(The working inside the brackets also follows BoDMAS, so 3×10 first, then -6)

Setting Out Your Working

It is important to be able to communicate in mathematics. You have to be able to explain to another person how you have arrived at your answer in a mathematical and concise way.

If you write an equals sign, the things that come before and after that sign must be equal to each other.

Look at how two students answer the same question, showing their working.

Rita writes: $(10 + 2) \div 4 = 10 + 2 = 12 \div 4 = 3$

Sara writes: $(10 + 2) \div 4$
$= 12 \div 4$
$= 3$

Which is the easier to follow?

In the first case, Rita has written $10 + 2 = 12 \div 4$. But this is not true!

Sara has set out her work so that the equals sign means exactly that. She has also used a new line between each bit of working, which makes it easier to read.

The examples throughout this book will show you how to set out your work, so do practise this right from the beginning. In general, writing one equals sign per line is good practice. However, please note that in this book, for reasons of economy and space, it has not always been possible to restrict working to one equals sign per line.

Exercise 1.6

NO CALCULATOR UNLESS SPECIFIED

1 Write in standard form:
a 12 000 **b** 365 **c** 59 103 **d** 6000 **e** 7 010 400

2 Write in standard form:
a 0.0035 **b** 0.156 **c** 0.0005 **d** 0.000 0043 **e** 0.0102

3 Write in standard form:
a 0.003 45 **b** 520 160 **c** 112
d 0.001 **e** 0.1001 **f** 2 million

4 Write as a normal number:
a 5.6×10^3 **b** 2.7×10^{-4} **c** 1.16×10^{-2} **d** 6×10^5 **e** 2×10^{-3}

5 Calculate the following, **without using a calculator**:
a $4 + 7 \times 2$ **b** $12 \div 3 \times 2 + 6$
c $1 + 2 + 3 - (2 \times 3)$ **d** $(4 + 5) \div (4 - 1)$

Check your answers with a calculator.

6 **Use your calculator** to work out the following:
a $(5 + 7 - 2) \div (6 - 4)$ **b** $2 \times 3 + 5 \times 7$ **c** $3 \times (14 - 7) - 2$

Check your answers by calculating without the calculator.

7 Put brackets in the right places to make each of these sums correct:
a $5 - 3 \times 4 = 8$ **b** $9 + 50 - 24 + 2 = 22$ **c** $31 - 15 \div 10 - 2 = 2$

Exercise 1.7

NO CALCULATOR IN THIS EXERCISE

Mixed exercise

1 **a** {–5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17}
Using the set of numbers above, answer true or false to the following:
i All the numbers come from the set of real numbers.
ii All the numbers come from the set of rational numbers.
iii All the numbers come from the set of natural numbers.
iv All the numbers come from the set of integers.

b Insert an inequality sign to make the following true:
i –4 … 3 **ii** 0 … –2 **iii** 5 … –5 **iv** 3 … –2

c List (in curly brackets):
i the set of prime numbers less than 10
ii the set of factors of 45
iii the set of multiples of 3 less than 20.

2 Find the LCM of:
a 12 and 20 **b** 5 and 15 and 90

3 Find the HCF of:
a 16 and 12 **b** 20 and 8 and 12

4 Calculate the following:
a 2.1^2 **b** 3^3 **c** $\sqrt{81}$ **d** $\sqrt{8100}$ **e** $\sqrt[3]{125}$

5 Write 600 as a product of its prime factors.
6 List all the factors of 160.

Exam-style questions

7 Tasnim records the temperature, in °C, at 6 a.m. every day for 10 days:
–6, –3, 0, –2, –1, –7, –5, 2, –1, –3
- **a** Find the difference between the highest and the lowest temperatures.
- **b** Find the median temperature. (4024 paper 12 Q2 June 2012)

8 Add brackets to the expression to make it correct.
$1 + 72 \div 4 \times 2 = 10$ (4024 paper 01 Q3b June 2012)

9 Work out $4^3 - 5^2$. (0580 paper 01 Q1 June 2004)

10 The Dead Sea shore is 395 metres **below** sea level. Hebron is 447 metres **above** sea level. Find the difference in height. (0580 paper 01 Q2 June 2004)

11 **a** Express 154 as the product of its prime factors.
- **b** Find the lowest common multiple of 154 and 49. (4024 paper 01 Q6 June 2007)

12 Place brackets in the following calculation to make it a correct statement.
$10 - 5 \times 9 + 3 = 60$ (0580 paper 01 Q2 November 2004)

13 Write down a multiple of 4 and 14 which is less than 30.
(0580 paper 01 Q1 November 2008)

14 Write 0.003 62 in standard form. (0580 paper 01 Q7 June 2008)

15 Written as the product of its prime factors, $360 = 2^3 \times 3^2 \times 5$.
- **a** Write 108 as the product of its prime factors.
- **b** Find the lowest common multiple of 108 and 360. Give your answer as the product of its prime factors.
- **c** Find the smallest positive integer k such that $360k$ is a cube number. (4024 paper 01 Q8 November 2006)

16 **a** Write down the two cube numbers between 10 and 100.
- **b** Write down the two prime numbers between 30 and 40. (4024 paper 01 Q3 June 2009)

17 **a** Write down all the factors of 18.
- **b** Write 392 as the product of its prime factors. (4024 paper 01 Q6 June 2009)

18 The numbers 294 and 784, written as the product of their prime factors, are
$294 = 2 \times 3 \times 7^2$ $784 = 2^4 \times 7^2$

Find
- **a** the largest integer which is a factor of both 294 and 784
- **b** $\sqrt{784}$. (4024 paper 01 Q4 November 2009)

Fractions, Decimals and Percentages

2

Learning Objectives (Syllabus sections 5, 6, 8, 12)

In this chapter you will:

- revise and learn more about fractions, decimals and percentages and, without using a calculator,
- convert between fractions, decimals and percentages
- work with fractions, decimals and percentages
- order quantities expressed as fractions, decimals and percentages.

2.1 Introduction

This chapter should give you the basic skills for working with fractions, decimals and percentages that you will need later in the course. You may already have a good grasp of the basic ideas, but misunderstandings and errors in the handling of fractions are often the cause of difficulties in arithmetic and algebra. Make sure you can complete the examples and exercises confidently.

You should **not** use a calculator when working through this chapter. It is important that you first understand the principles so that you will be able to work more easily with algebra. We will go on to more difficult work requiring a calculator in a later chapter.

Remember: no calculator in this chapter!

2.2 Essential Skills NO CALCULATOR IN THIS EXERCISE

Make sure you can calculate the following. Look back to the previous chapter if you need a reminder.

1 Find the LCM of the following numbers:

a 2, 5 **b** 7, 14 **c** 3, 8, 12 **d** 3, 5, 12, 60

2 Find the HCF of the following numbers:

a 12, 36 **b** 18, 24 **c** 50, 150, 200 **d** 40, 24, 56

2.3 Understanding Common Fractions

Key term

Common or vulgar fractions are ordinary fractions, for example $\frac{2}{3}$, usually just abbreviated to 'fractions'. Here, 2 is the **numerator**, 3 is the **denominator**.

When we use the word fraction we normally think of numbers like $\frac{7}{8}$, $\frac{2}{3}$ or $\frac{1}{2}$.

These are actually *common* or *vulgar* fractions.

In your O Level course the word fraction will normally mean common fraction, but sometimes it will help you to understand your work if you remember that decimals (decimal fractions) and percentages are also fractions. Percentages are fractions with a denominator of a hundred. For example, 21% is the same as $\frac{21}{100}$.

As you know, common fractions have a number above the fraction line, and another number below the fraction line. These numbers are called the **numerator** and the **denominator** respectively. So in the fraction $\frac{2}{3}$, 2 is the numerator and 3 is the denominator.

You can think of the denominator as the name of the fraction with the numerator showing the number of fractions with this name. Figure 2.1 should help you to see this.

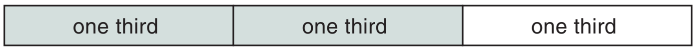


Figure 2.1 Two thirds

The strip in Figure 2.1 has been divided into three equal parts.

Each part is one third $\left(\frac{1}{3}\right)$ of the whole strip.

Three thirds $\left(\frac{3}{3}\right)$ make up the whole strip.

Two thirds $\left(\frac{2}{3}\right)$ are shaded.

The **numerator** is the top number in a common fraction.

The **denominator** is the bottom number in a common fraction.

The denominator shows into how many equal parts the whole strip has been divided. The denominator tells us the name of the fraction, in this case 'thirds'.

The numerator shows the number of these fractions, in the case 2 'thirds' have been shaded. Look at Figure 2.2 to see this drawn out.

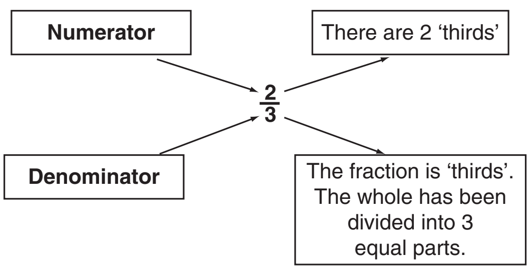


Figure 2.2 Common fraction

Mixed Numbers and Improper Fractions

Key term

Mixed numbers combine integers and fractions, for example $3\frac{2}{5}$. Here, 3 is the whole number part and $\frac{2}{5}$ is the fraction part.

Mixed numbers have a whole part and a fraction part. The mixed number $1\frac{2}{3}$ means there is one whole part and 2 thirds. Figure 2.3 shows two strips, each divided into three equal parts.

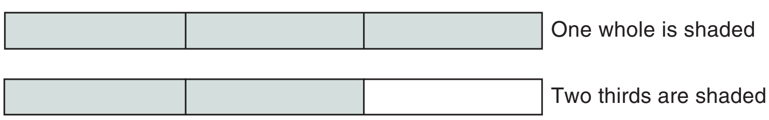


Figure 2.3 One and two thirds

NOTE:
An improper fraction is often referred to as a 'top heavy' fraction, which describes it well because the top number is larger than the lower number.

Key terms

Improper fractions are 'top heavy' fractions, for example $\frac{9}{5}$.

Equivalent fractions represent the same number, for example $\frac{3}{5}, \frac{6}{10}, \frac{90}{150}$ all represent $\frac{3}{5}$ of the whole.

Figure 2.3 also shows how we can write a mixed number as an improper fraction. An **improper** fraction is a mixed number written entirely in fractions, so the numerator is larger than the denominator. The diagram shows the shaded parts of the two strips as either $1\frac{2}{3}$ or $\frac{5}{3}$ (or 5 thirds).

Equivalent Fractions

Fractions can be given different names, and if the rules for doing this are followed, the resulting fraction is of the same size as the original.

Equivalent fractions are fractions of the same size, but with different denominators (names) and numerators. Look at Figure 2.4.

Figure 2.4 Two thirds

Figure 2.4 shows the strip divided into three equal parts with the fraction $\frac{2}{3}$ shaded as before. If we divide *each* third into two equal parts you should see that there are now six equal parts, and four of these are equivalent in size to 2 thirds. Figure 2.5 shows this.

Figure 2.5 Four sixths

This shows that $\frac{2}{3} = \frac{4}{6}$. These two fractions are called equivalent fractions because they represent the same amount of the whole strip. The rule for finding equivalent fractions is that the denominator *and* numerator have to be multiplied (or divided) by the same number. In this case the first fraction has had the numerator and denominator multiplied by 2. You will find out more about this later.

More Examples of Fractions

We can work with things other than strips of paper to understand fractions. Imagine a bag containing 20 sweets. You want to share these sweets equally among four people. The 20 sweets would have to be divided into 4 equal parts. There would be 5 sweets in each part. Each part would be one quarter of the whole.

This could be written as $\frac{1}{4} \times 20 = 5$, as shown in Figure 2.6.

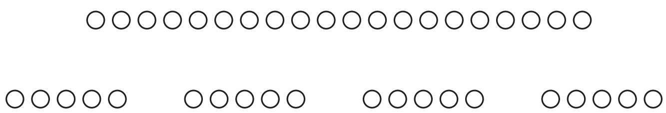


Figure 2.6 Twenty sweets

How many counters would there be in $\frac{2}{3}$ of 15 counters?

You can use the diagram of 15 counters in Figure 2.7 to help.

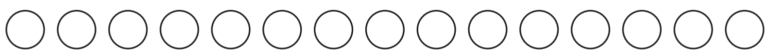


Figure 2.7 Fifteen counters

Look at the clock face in Figure 2.8.

We know that 15 minutes is a quarter of an hour, and that there are 60 minutes in one hour. The hour is divided into sixty equal parts. So each minute is 1 sixtieth $\left(\frac{1}{60}\right)$ of an hour. Therefore, fifteen minutes is fifteen sixtieths of one hour.

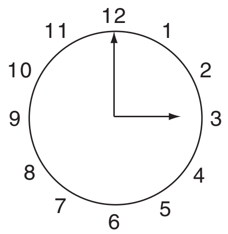


Figure 2.8 Fractions of an hour

Simplifying shows that $\frac{15}{60} = \frac{1}{4}$ (divide numerator and denominator by 15).

How do we work out what fraction of an hour is ten minutes?

Write ten sixtieths and simplify.

$\frac{10}{60} = \frac{1}{6}$ (divide numerator and denominator by 10).

So ten minutes is one sixth of an hour.

Other shapes can be divided into equal parts.

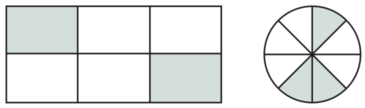


Figure 2.9 Other representations

The rectangle in Figure 2.9 has been divided into six equal areas, and so each is $\frac{1}{6}$th of the rectangle.

Two of these are shaded. This means that $\frac{2}{6}$, which is equivalent to $\frac{1}{3}$ of the rectangle, is shaded. The circle has been divided into 8 equal parts, and 3 are shaded, so $\frac{3}{8}$ of the circle is shaded.

2.4 Working with Common Fractions

Changing a Mixed Number to an Improper Fraction

As an example, the steps to be followed to change $3\frac{1}{5}$ to an improper fraction are given below:

- multiply: $3 \times 5 = 15$ (there are 15 fifths in three wholes, so $3\frac{1}{5} = \frac{15}{5} + \frac{1}{5}$)
- add: $15 + 1 = 16$ (add the extra 1 fifth)
- answer: $\frac{16}{5}$ (16 fifths).

Changing an Improper Fraction to a Mixed Number

As an example, the steps to be followed to change $\frac{23}{4}$ to a mixed number are given below:

- divide: $23 \div 4 = 5$ remainder 3 (23 quarters = 5 wholes with 3 quarters left over)
- answer: $5\frac{3}{4}$.

Equivalent Fractions

As an example, to change $\frac{4}{10}$ to equivalent fractions:

- either multiply numerator and denominator by the same number: $\frac{4 \times 2}{10 \times 2} = \frac{8}{20}$
- or divide numerator and denominator by the same number: $\frac{4 \div 2}{10 \div 2} = \frac{2}{5}$.

Addition and Subtraction of Fractions

Before fractions are added or subtracted, we have to make sure they have the same name. For example, look at Figure 2.10, which represents the addition sum $\frac{3}{4} + \frac{1}{8}$. The only way we can add these two is to write them with the same name (denominator).

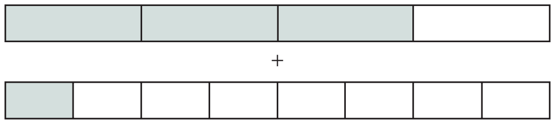


Figure 2.10 Three quarters plus one eighth

To do this we divide *each* of the quarters into two equal parts, to make eighths. The three quarters has become six eighths and can now be added to the one eighth, as in Figure 2.11.

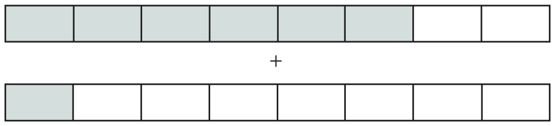


Figure 2.11 Six eighths plus one eighth

It is now easy to see that $\frac{3}{4}+\frac{1}{8}=\frac{6}{8}+\frac{1}{8}=\frac{7}{8}$.

The answer is seven eighths.

It can be easier to add or subtract mixed numbers by changing them to top heavy (improper) fractions first, as you will see in Example 1, part **f iv**.

You may have to change both fractions to equivalent fractions with the same denominator.

For example, consider $\frac{2}{3}+\frac{4}{5}$.

Follow these steps to see how to work this out:

- Change any mixed numbers to improper fractions.
- Find the lowest common multiple of both denominators (LCM of 3 and 5 is 15).
- Change both fractions to equivalent fractions with the same denominator $\left(\frac{2\times5}{3\times5}+\frac{4\times3}{5\times3}=\frac{10}{15}+\frac{12}{15}\right)$.
- Add or subtract the fractions in the usual way $\left(\frac{10}{15}+\frac{12}{15}=\frac{22}{15}\right)$.
- Simplify and change to a mixed number if necessary $\left(\frac{22}{15}=1\frac{7}{15}\right)$.
- Answer: $\frac{2}{3}+\frac{4}{5}=1\frac{7}{15}$.

Simplifying Fractions

Simplifying fractions refers to writing them in the simplest equivalent form. For example, $\frac{4}{10}$ can be simplified by dividing both the numerator *and* denominator by 2. This means that $\frac{4}{10}=\frac{2}{5}$.

This is often called **'cancelling down'** the fraction.

For example, the steps to be followed to simplify $\frac{42}{162}$ are given below.

Either:

- Find any common factor and divide the numerator and denominator by this number:

$$\frac{42\div2}{162\div2}=\frac{21}{81}$$

- Repeat if possible: $\frac{21\div3}{81\div3}=\frac{7}{27}$.
- Stop when there are no more common factors.
- Answer: $\frac{7}{27}$.

Or:

- Find the HCF of the numerator and denominator to simplify in one step:

$$\frac{42\div6}{162\div6}=\frac{7}{27}$$

Key terms

Simplifying fractions means expressing them in their lowest terms, for example $\frac{20}{35}$ simplifies to $\frac{4}{7}$.

Cancelling down is writing a fraction in a simpler form by dividing the numerator and denominator by the same number.

Example 1

a Change the top heavy (improper) fraction $\frac{7}{2}$ to a mixed number.

b Change the mixed number $4\frac{3}{5}$ to an improper fraction.

c Which of these fractions are equivalent?

$\frac{20}{50}, \frac{2}{5}, \frac{3}{10}, \frac{4}{10}, \frac{3}{5}, \frac{8}{20}$

d Change $\frac{4}{5}$ to twentieths.

e Fill in the blank spaces to give equivalent fractions.

$\frac{\_\_}{16} = \frac{3}{8} = \frac{30}{\_\_} = \frac{15}{\_\_}$

f Calculate, simplifying and writing the answers as mixed numbers if necessary:

i $3 + \frac{5}{6}$ **ii** $\frac{5}{8} + \frac{1}{2}$ **iii** $\frac{3}{4} - \frac{2}{3}$

iv $2\frac{1}{3} + 4\frac{5}{6}$ **v** $1 - \frac{7}{9}$

g Write each of the following fractions in their simplest forms:

i $\frac{5}{40}$ **ii** $\frac{6}{48}$ **iii** $\frac{18}{72}$

h How many sheep are there in 3 fifths of a flock of 25 sheep?

i How many students are there in $\frac{2}{7}$ of a class of 35?

Answer 1

a $\frac{7}{2} = 3\frac{1}{2}$ **b** $4\frac{3}{5} = \frac{23}{5}$ **c** $\frac{20}{50} = \frac{2}{5} = \frac{4}{10} = \frac{8}{20}$

d $\frac{4}{5} = \frac{4 \times 4}{5 \times 4} = \frac{16}{20}$ **e** $\frac{6}{16} = \frac{3}{8} = \frac{30}{80} = \frac{15}{40}$

f **i** $3 + \frac{5}{6} = 3\frac{5}{6}$ **ii** $\frac{5}{8} + \frac{1}{2} = \frac{5}{8} + \frac{4}{8} = \frac{9}{8} = 1\frac{1}{8}$

iii $\frac{3}{4} - \frac{2}{3} = \frac{3 \times 3}{4 \times 3} - \frac{2 \times 4}{3 \times 4} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$ **iv** $2\frac{1}{3} + 4\frac{5}{6} = \frac{7}{3} + \frac{29}{6} = \frac{14}{6} + \frac{29}{6} = \frac{43}{6} = 7\frac{1}{6}$

v $1 - \frac{7}{9} = \frac{9}{9} - \frac{7}{9} = \frac{2}{9}$

g **i** $\frac{5}{40} = \frac{5 \div 5}{40 \div 5} = \frac{1}{8}$ **ii** $\frac{6}{48} = \frac{1}{8}$ **iii** $\frac{18}{72} = \frac{18 \div 2}{72 \div 2} = \frac{9}{36} = \frac{1}{4}$

h One fifth of the flock is 5 sheep, so 3 fifths is 15 sheep.

i $\frac{1}{7}$ of $35 = 5$, so $\frac{2}{7}$ of $35 = 10$. Answer: 10 students.

Exercise 2.1

NO CALCULATOR IN THIS EXERCISE

1 Change to mixed numbers:

a $\frac{19}{5}$ **b** $\frac{201}{10}$ **c** $\frac{33}{2}$

2 Change to improper fractions:

a $3\frac{7}{8}$ **b** $100\frac{1}{2}$ **c** $3\frac{11}{12}$

3 Fill in the blank spaces to give equivalent fractions:

$\frac{5}{\_\_} = \frac{10}{30} = \frac{\_\_}{3} = \frac{7}{\_\_} = \frac{21}{\_\_}$

4 Write the following as hundredths (denominator = 100):

a $\frac{7}{10}$ **b** $\frac{4}{25}$ **c** $\frac{19}{20}$

d $\frac{52}{200}$ **e** $\frac{81}{900}$

5 Calculate the following, simplifying and writing your answers as mixed numbers if necessary:

a $\frac{3}{7} + \frac{2}{7}$ **b** $\frac{4}{5} - \frac{3}{5}$ **c** $\frac{7}{12} - \frac{1}{6}$ **d** $\frac{2}{9} + \frac{3}{4}$

e $2\frac{1}{5} + 1\frac{3}{4}$ **f** $3\frac{2}{5} - 1\frac{1}{2}$ **g** $1 - \frac{6}{7}$ **h** $1 - \frac{5}{12}$

6 Simplify:

a $\frac{22}{77}$ **b** $\frac{60}{72}$ **c** $\frac{45}{60}$ **d** $\frac{45}{360}$

7 How many sweets would be in a bag of 28 sweets after $\frac{1}{4}$ of them had been eaten?

8 One third of a class of 45 students has gone away on a field trip. How many students have gone on the trip?

Multiplying and Dividing Fractions

The first part of Figure 2.12 shows a strip divided into thirds, with one third shaded.

We can use this figure to work out $\frac{1}{2}\times\frac{1}{3}$, which means $\frac{1}{2}$ **of** $\frac{1}{3}$.

The second part of Figure 2.12 shows the same strip with the shaded third divided into two equal parts. Each of these is one half of a third of the strip.

You should see that each of these is equal to one sixth of the whole strip.

So $\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$, which means $\frac{1}{2}$ of $\frac{1}{3}=\frac{1}{6}$.

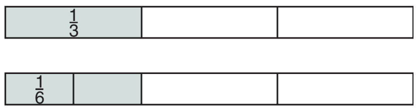


Figure 2.12 Multiplying fractions

You will probably find multiplying and dividing fractions easier than adding and subtracting.

The rules for multiplying fractions are:

- Change any mixed numbers to top heavy (improper) fractions.
- Write any whole numbers over one.
- Multiply the numerators together, and multiply the denominators together.
- Simplify the answer if necessary, and change to a mixed number if necessary.

Applying these rules to our example above:

$$\frac{1}{2}\times\frac{1}{3}=\frac{1\times1}{2\times3}=\frac{1}{6}$$

Example 2

a Multiply the following fractions, simplifying and writing your answers as mixed numbers if necessary:

i $3\times\frac{3}{4}$ **ii** $\frac{5}{6}\times\frac{2}{3}$ **iii** $\frac{6}{7}\times2\frac{1}{3}$ **iv** $1\frac{2}{3}\times2\frac{1}{5}$

b Calculate the following:

i $\left(\frac{3}{5}\right)^2$ **ii** $\left(\frac{2}{3}\right)^3$

Answer 2

a **i** $3\times\frac{3}{4}=\frac{3}{1}\times\frac{3}{4}=\frac{9}{4}=2\frac{1}{4}$ **ii** $\frac{5}{6}\times\frac{2}{3}=\frac{10}{18}=\frac{5}{9}$

iii $\frac{6}{7}\times2\frac{1}{3}=\frac{6}{7}\times\frac{7}{3}=\frac{42}{21}=\frac{2}{1}=2$ **iv** $1\frac{2}{3}\times2\frac{1}{5}=\frac{5}{3}\times\frac{11}{5}=\frac{55}{15}=\frac{11}{3}=3\frac{2}{3}$

b **i** $\left(\frac{3}{5}\right)^2=\frac{3}{5}\times\frac{3}{5}=\frac{9}{25}$ **ii** $\left(\frac{2}{3}\right)^3=\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}=\frac{8}{27}$

You have probably noticed that in Example 2**a** parts **ii**, **iii** and **iv** the working could have been shortened considerably by simplifying earlier. We will look at this now.

In Example 2**a ii**:

$$\frac{5}{6}\times\frac{2}{3}=\frac{5\times 2}{6\times 3}=\frac{5\times 2\div 2}{6\times 3\div 2}=\frac{5}{9}$$

So we could have simplified before doing the multiplication:

$\frac{5}{6}\times\frac{2}{3}=\frac{5}{3\times 3}=\frac{5}{9}$ (dividing the top and bottom of the fraction by 2)

In Example 2**a iii**:

$\frac{6}{7}\times 2\frac{1}{3}=\frac{6}{7}\times\frac{7}{3}=\frac{6}{3}=2$ (divide top and bottom by 7 first, and then by 3)

Try Example 2(a) (iv) yourself.

Warning: This only works for multiplication, so do not use it in addition or subtraction!

How can we visualise division? Remember that if you do the division $10\div 2$ you are finding how many twos there are in ten. The answer of course is 5.

Think about $\frac{3}{4}\div\frac{1}{8}$. This means 'how many eighths are there in three quarters?'

Figure 2.13 shows one strip with $\frac{3}{4}$ shaded, and another divided into eight equal parts and shaded to show that 6 eighths will go exactly into $\frac{3}{4}$. So the answer is 6.

i.e. $\frac{3}{4}\div\frac{1}{8}=6$

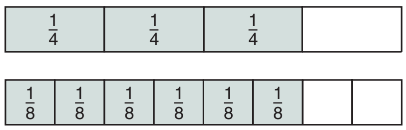


Figure 2.13 Dividing fractions

The rules for dividing fractions are:

- Change any mixed numbers to top heavy (improper) fractions.
- Write any whole numbers over one.
- Change the division sign to multiplication.
- Turn the second fraction upside down.
- Proceed as for multiplication.

Using these rules for $\frac{3}{4}\div\frac{1}{8}$, we get:

$$\frac{3}{4}\div\frac{1}{8}=\frac{3}{4}\times\frac{8}{1}=\frac{24}{4}=6$$

Example 3

Do the following divisions:

a $\frac{5}{6}\div 3$ **b** $\frac{3}{4}\div\frac{1}{2}$ **c** $1\frac{2}{5}\div 4\frac{3}{5}$ **d** $\frac{2}{5}\div\frac{5}{8}$

Answer 3

a $\frac{5}{6}\div 3=\frac{5}{6}\div\frac{3}{1}=\frac{5}{6}\times\frac{1}{3}=\frac{5}{18}$

b $\frac{3}{4}\div\frac{1}{2}=\frac{3}{4}\times\frac{2}{1}=\frac{6}{4}=\frac{3}{2}=1\frac{1}{2}$ (or $\frac{3}{4}\times\frac{2}{1}=\frac{3}{2}=1\frac{1}{2}$ by dividing top and bottom by 2)

c $1\frac{2}{5}\div 4\frac{3}{5}=\frac{7}{5}\div\frac{23}{5}=\frac{7}{5}\times\frac{5}{23}=\frac{7}{23}$ (by dividing top and bottom by 5)

d $\frac{2}{5}\div\frac{5}{8}=\frac{2}{5}\times\frac{8}{5}=\frac{16}{25}$ (Be careful! You cannot divide top and bottom by 5 here!)

Looking at these examples you should see that you can do the simplifying shortcut *only after* the second fraction has been inverted and the sign has changed from division to multiplication.

Exercise 2.2

NO CALCULATOR IN THIS EXERCISE

Calculate the following, simplifying your answers and changing to mixed numbers as necessary:

1	**a** $3\times\frac{1}{5}$		**b** $3\times\frac{2}{5}$		**c** $\frac{3}{4}\times 10$	
2	**a** $\frac{1}{3}\times\frac{1}{6}$		**b** $\frac{3}{4}\times\frac{1}{7}$		**c** $\frac{5}{8}\times\frac{3}{4}$	
3	**a** $\frac{5}{7}\times\frac{1}{10}$		**b** $\frac{7}{8}\times\frac{2}{5}$		**c** $\frac{4}{9}\times\frac{3}{8}$	
4	**a** $2\frac{1}{2}\times 3\frac{1}{4}$		**b** $5\frac{3}{5}\times 2\frac{1}{8}$		**c** $1\frac{1}{3}\times 2\frac{1}{3}$	
5	**a** $3\div\frac{2}{5}$		**b** $\frac{4}{3}\div 3$		**c** $\frac{2}{7}\div 4$	
6	**a** $\frac{1}{2}\div\frac{3}{7}$		**b** $\frac{3}{7}\div\frac{1}{2}$		**c** $\frac{5}{9}\div\frac{6}{7}$	
7	**a** $\frac{2}{9}\div\frac{2}{3}$		**b** $\frac{2}{3}\div\frac{2}{9}$		**c** $\frac{5}{8}\div\frac{3}{8}$	
8	**a** $2\frac{3}{4}\div 1\frac{1}{2}$		**b** $5\frac{1}{2}\div\frac{3}{4}$		**c** $3\frac{3}{5}\div 1\frac{2}{3}$	
9	**a** $\frac{2}{5}\div\frac{1}{15}$		**b** $\frac{3}{5}\div\frac{5}{6}$		**c** $2\frac{1}{3}\div\frac{3}{7}$	

2.5 Working with Decimals

Key term

Decimal fractions are usually abbreviated to 'decimals'.

Decimals (or decimal fractions) are usually easier to work with than (common) fractions, so the rules and a few examples should be sufficient to remind you how to do each operation. We will abbreviate to decimals and fractions because these terms are generally understood to mean decimal fractions and common fractions.

Addition and Subtraction of Decimals

As an example, to simplify 3 + 1.205 + 40.016 follow the steps given below:

- Add '.0' to the whole number to remind you where the decimal point belongs. (3.0 + 1.205 + 40.016)
- Write the numbers in column form, but with the decimal points in a vertical line.

$$\begin{array}{r} 3.0 \\ 1.205+ \\ 40.016+ \end{array}$$

- Starting from the right, add (or subtract) using the normal methods of addition (or subtraction).
- Place the decimal point in the answer vertically under the other decimal points.

$$\begin{array}{r} 3.0 \\ 1.205+ \\ \underline{40.016}+ \\ 44.221 \end{array}$$

Multiplying Decimals

As an example, to simplify 2.16×0.002 follow the steps given below:

- At first, ignore the decimal points.
- Starting from the right multiply using the normal methods.
- Count how many digits (numbers) come after the decimal points.
- Starting from the right count back this number of places and insert the decimal point, inserting zeroes if necessary.

$$\begin{array}{r} 2.16 \\ 0.002\times \\ \hline 0.00432 \end{array}$$

(There are 5 digits after the decimal points, so counting 5 places from the right it is necessary to insert 2 zeroes.)

- Answer: $2.16 \times 0.002 = 0.00432$.

NOTE:
Remember that *multiplying* by 10, 100 and so on will make the answer *larger*.

Multiplying by 10, 100, 1000 and so on is straightforward with decimals. For example, 0.013×100:

- Count the number of zeroes (2 in this example) in the number you are multiplying by.
- Move the decimal point to the *right* by the same number of places, inserting zeroes if necessary ($0.013 \times 100 = 1.3$).
- Answer: $0.013 \times 100 = 1.3$.

Dividing Decimals

As an example, to divide 63.6 by 0.012 follow the steps given below:

- Write the first number over the second number $\left(\frac{63.6}{0.012}\right)$.
- Multiply top and bottom by 10, 100, 1000 or 10 000 until the lower number is a whole number (in this case we need to use 1000, i.e. $\frac{63.6\times1000}{0.012\times1000} = \frac{63600}{12}$).
- Divide the new lower number into the top number ($63600 \div 12 = 5300$).
- Answer: 63.6 divided by $0.012 = 5300$.

NOTE:
Remember that dividing by 10, 100, 1000 and so on will make the number smaller.

Dividing decimals by 10, 100, 1000 and so on is also straightforward. For example, divide 0.234 by 1000:

- Count the number of zeroes (three in this case) in the number you are to divide by.
- Move the decimal point 3 places *left*, filling in zeroes if necessary ($0.234 \div 1000 = 0.000234$).
- Answer: $0.234 \div 1000 = 0.000234$.

Example 4

Calculate the following:

a $2.501 + 12.6$ **b** $45.3173 - 1.012$ **c** 3.513×100

d 0.012×10 **e** 4.12×1000 **f** 2.1×1.1

g $0.16 \div 100$ **h** $31.323 \div 0.03$

Answer 4

a $2.501 + 12.6 = \begin{array}{r} 2.501 \\ 12.6+ \\ \hline 15.101 \end{array}$

b $45.3173 - 1.012 = \begin{array}{r} 45.3173 \\ 1.012- \\ \hline 44.3053 \end{array}$

c $3.513 \times 100 = 351.3$

d $0.012 \times 10 = 0.12$

e $4.12 \times 1000 = 4120$

f $2.1 \times 1.1 = \begin{array}{r} 2.1 \\ 1.1\times \\ \hline 2.31 \end{array}$

g $0.16 \div 100 = 0.0016$

h $31.323 \div 0.03 = \frac{31.323\times100}{0.03\times100} = \frac{3132.3}{3} = 1044.1$

Exercise 2.3

NO CALCULATOR IN THIS EXERCISE

Calculate the following.

1 $3.5 + 0.16 + 10.2$ **2** $501 + 1.67 + 0.3$ **3** $17.95 - 1.4$

4 $6.119 - 2.01$ **5** 13.41×1000 **6** 0.0169×1000

7 $6.017 \div 100$ **8** 10.2×3.1 **9** $18.96 \div 1.2$

2.6 Percentages

Key term

Percentages are fractions with a denominator of 100. Think of the % sign as 'out of 100'.

It may help you to visualise **percentages** and compare them with fractions if you imagine a stack of, say 100 counters, as in Figure 2.14. Imagine that the counters are numbered from 1 to 100, with 1 at the bottom of the stack.

Each of the counters is $\frac{1}{100}$ of the whole stack, so each counter is 1% of the stack. The whole stack is 100% of the stack or one whole.

Now you can see that half way up is 50%, one quarter of the way up is 25%, $\frac{1}{10}$ of the way up is 10% and so on.

Copy Figure 2.14 and mark in $\frac{3}{4}$ and its corresponding percentage, 20% and any others that you can think of.

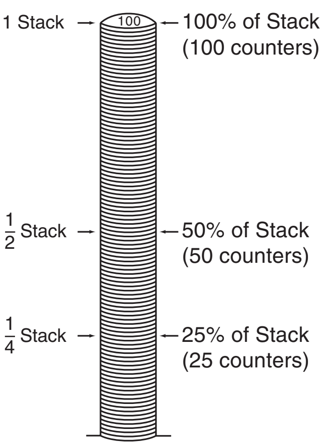


Figure 2.14 Percentages

2.7 Conversion Between Common Fractions, Decimals and Percentages

Common fraction	**Divide the numerator by the denominator**	**Decimal fraction**
$\frac{2}{5}$	2 divided by 5	0.4
$3\frac{2}{5}$		3.4
Decimal fraction	**Multiply by 100**	**Percentage**
0.45	0.45×100	45%
0.613	0.613×100	61.3%
2.051	2.051×100	205.1%

Divide by 100 to change a percentage to decimal fraction.

Decimal fraction	**Write decimal over 1, then multiply the top and bottom by 10, or 100, or 1000 until the numerator is a whole number. Simplify if necessary.**	**Common fraction**
0.7	$\frac{0.7 \times 10}{1 \times 10}$	$\frac{7}{10}$
0.45	$\frac{0.45 \times 100}{1 \times 100} = \frac{45}{100}$	$\frac{45}{100} = \frac{9}{20}$

Exercise 2.4

NO CALCULATOR IN THIS EXERCISE

Copy and complete Table 2.1 for conversions between common fractions, decimals and percentages. It is a good idea to learn these as they occur quite frequently and you can save time if you know them. The last two have been done for you, and it is *definitely* a good idea to learn these!

The dot above a number means that the number repeats forever. For example, $0.\dot{3}$ means 0.33333333333…; it is called 'zero point three recurring'.

	Fraction	Decimal	Percentage
1	$\frac{1}{2}$		
2		0.25	
3			75%
4	$\frac{1}{10}$		
5		0.3	
6		0.2	
7			12.5%
8	$\frac{1}{3}$	$0.\dot{3}$	$33\frac{1}{3}\%$
9	$\frac{2}{3}$	$0.\dot{6}$	$66\frac{2}{3}\%$

Table 2.1 Converting between fractions, decimals and percentages

Calculating Percentages of an Amount

There are several ways to calculate percentages quickly.

The first is to know the common percentages (50%, 10%, 25% and so on) and their corresponding fractions (see part **a** of Example 5).

The second is to find 1% by dividing by 100, and then multiply by whatever percentage is required (see part **b** of Example 5).

Lastly, some percentages can be 'built up' from smaller percentages that are easy to find (see part **c** of Example 5).

Example 5

a Find:

i 50% of 136 **ii** 10% of 34 **iii** 75% of 32

b Find:

i 7% of 61 **ii** 21% of 400 **iii** 12% of 700

c Find:

i 15% of 96 **ii** 65% of 140 **iii** 17.5% of 260

Answer 5

a **i** 50% of $136 = \frac{1}{2}$ of $136 = 68$ **ii** 10% of $34 = \frac{1}{10}$ of $34 = 3.4$

iii 75% of $32 = \frac{3}{4}$ of $32 = 3 \times \frac{1}{4}$ of $32 = 3 \times 8 = 24$

b **i** 7% of $61 = 7 \times 1\%$ of $61 = 7 \times \frac{1}{100}$ of $61 = 7 \times 0.61 = 4.27$

ii 21% of $400 = 21 \times 1\%$ of $400 = 21 \times \frac{1}{100}$ of $400 = 21 \times 4 = 84$

iii 12% of $700 = 12 \times 7 = 84$

c **i** To find 15% of 96:

find 10% of 96 ($\frac{1}{10}$ of 96) = 9.6

find 5% of 96 ($\frac{1}{2}$ of 10%) = 4.8

Add 10% and 5%: 15% of 96 = 9.6 + 4.8 = 14.4

ii 65% of 140:

50% of 140 = 70

10% of 140 = 14 + (one fifth of 50%)

5% of 140 = 7 + (half of 10%)

65% of 140 = 91

Answer: 65% of 140 = 91

iii 17.5% of 260:

10% of 260 = 26

5% of 260 = 13 + (half of 10%)

2.5% of 260 = 6.5 + (half of 5%)

17.5% of 260 = 45.5

Answer: 17.5% of 260 = 45.5

Exercise 2.5

NO CALCULATOR IN THIS EXERCISE

Calculate the following, showing your method:

1 75% of 64

2 30% of 1550

3 9% of 3400

4 55.5% of 680

5 3% of 73

Finding One Number as a Percentage of Another

We sometimes need to express one number as a percentage of another. For example, you get 6 answers correct out of 20 in a test. What is your percentage mark?

- First make a fraction by writing the first number over the second $\left(\frac{6}{20}\right)$.
- Change the fraction to a percentage by multiplying by 100 over 1.

$$\left(\frac{6}{20} \times \frac{100}{1} = \frac{600}{20} = 30\%\right)$$

There is an alternative method that can sometimes be used, if the denominator of the fraction is a factor of 100:

- First make the fraction as before $\left(\frac{6}{20}\right)$.
- Change to the equivalent fraction with the denominator as 100.

$$\left(\frac{6 \times 5}{20 \times 5} = \frac{30}{100} = 30\%\right)$$

Example 6

a Find 25 as a percentage of 40. **b** Find 15 as a percentage of 25.

Answer 6

a $\frac{25}{40}\times\frac{100}{1}=\frac{250}{4}=62.5\%$ **b** $\frac{15\times4}{25\times4}=\frac{60}{100}=60\%$

Exercise 2.6

NO CALCULATOR IN THIS EXERCISE

Calculate the first number as a percentage of the second:

1 35, 140 **2** 72, 600 **3** 23, 50

4 40, 125 **5** 17, 250 **6** 90, 180

7 12, 6 **8** 29, 1000

2.8 Ordering Quantities

It is often easiest when comparing fractions, decimals and percentages to change them all to decimals. Alternatively, compare fractions by finding equivalent fractions with the same denominator.

Example 7

a Using the symbols $>$, $<$ or $=$, insert the correct sign to make the following statements true:

i 22 … 21 **ii** 0.75 … $\frac{3}{4}$ **iii** 0.25 … 25 **iv** $\frac{1}{3}$ … 0.3

b Write the following in order of size, starting with the smallest:

i 0.48, 0.408, 0.390, 0.399 **ii** $\frac{2}{5}, \frac{3}{5}, \frac{3}{10}, \frac{9}{20}$

iii 33%, 0.5, $\frac{3}{10}, \frac{1}{3}$

c Rafi loves eating naan. Do you think he would rather have two thirds or three quarters of a naan? Why?

d Find a fraction which is between each of the following pairs:

i $\frac{7}{10}$ and $\frac{9}{10}$ **ii** $\frac{1}{2}$ and $\frac{3}{4}$ **iii** $\frac{6}{8}$ and $\frac{7}{8}$

Answer 7

a **i** $22>21$ **ii** $0.75=\frac{3}{4}$ **iii** $0.25<25$ **iv** $\frac{1}{3}>0.3$

b **i** $0.390<0.399<0.408<0.48$

ii Changing $\frac{2}{5}, \frac{3}{5}, \frac{3}{10}, \frac{9}{20}$ to twentieths

$\frac{8}{20}, \frac{12}{20}, \frac{6}{20}, \frac{9}{20}$

and putting in order,

$\frac{6}{20}, \frac{8}{20}, \frac{9}{20}, \frac{12}{20}$

simplifying again,

$\frac{3}{10}<\frac{2}{5}<\frac{9}{20}<\frac{3}{5}$

iii Changing 33%, 0.5, $\frac{3}{10}, \frac{1}{3}$ to decimals,

0.33, 0.5, 0.3, 0.3333…

putting in order,

0.3, 0.33, 0.3333…, 0.5,

and re-writing as before $\frac{3}{10}<33\%<\frac{1}{3}<0.5$

c $\frac{3}{4} = 75\%$ and $\frac{2}{3} = 66\frac{2}{3}\%$, so Rafi would rather have $\frac{3}{4}$ of the naan.

d i $\frac{8}{10}$ is between $\frac{7}{10}$ and $\frac{9}{10}$; $\frac{8}{10} = \frac{4}{5}$.

Answer: $\frac{4}{5}$

iii $\frac{1}{2}$ and $\frac{3}{4}$ can be changed to their equivalent fractions $\frac{4}{8}$ and $\frac{6}{8}$, so $\frac{5}{8}$ is between $\frac{1}{2}$ and $\frac{3}{4}$.

Answer: $\frac{5}{8}$

iii $\frac{6}{8}$ and $\frac{7}{8}$ are equivalent to $\frac{12}{16}$ and $\frac{14}{16}$, so

Answer: $\frac{13}{16}$

Exercise 2.7

NO CALCULATOR IN THIS EXERCISE

1 Find a fraction that lies between $\frac{3}{5}$ and $\frac{4}{5}$.

2 Place the following in order of size, starting with the smallest:

451, 4.579, 4.098, 4.105

3 Place these fractions in order of size, starting with the smallest:
$\frac{4}{3}, \frac{2}{3}, \frac{3}{4}, \frac{17}{20}$

4 Place the following in order of size, starting with the smallest:
$\frac{33}{100}, 33\frac{1}{3}\%, \frac{3}{25}, \frac{3}{50}, \frac{67}{200}$

Exercise 2.8

NO CALCULATOR IN THIS EXERCISE

Exam-style questions

NOTE:
'Evaluate' means 'work out a numerical answer'.

1 **a** Evaluate $3\frac{2}{3} - 2\frac{4}{5}$. **b** Express $\frac{48}{84}$ in its lowest terms.

(4024 paper 11 Q2 November 2011)

2 **a** Add brackets to the equation to make it correct.
$4 + 6 \times 7 - 5 = 16$

b Find the value of 27×0.002. (4024 paper 01 Q2 November 2009)

3 **a** Express $\frac{13}{20}$ as a decimal.

b In a test, Rose scored 56 marks out of 70.
Express this score as a percentage. (4024 paper 01 Q1 June 2005)

4 Evaluate

a $2\frac{2}{3} \times \frac{1}{7}$, **b** $\frac{2}{5} \div \frac{7}{12}$. (4024 paper 01 Q2 June 2005)

5 **a** Express 0.527 as a percentage.

b Evaluate $5.6 \div 0.08$. (4024 paper 01 Q1 June 2006)

6 Evaluate

a $\frac{6}{7}-\frac{1}{3}$, **b** $\frac{2}{5}\times\frac{4}{9}$. (4024 paper 01 Q2 June 2006)

7 In an examination, Alan obtained 32 out of 40 marks. In another examination Ben obtained $\frac{5}{8}$ of the total marks.
Express the mark of each candidate as a percentage. (4024 paper 01 Q4 June 2006)

8 Evaluate
a $\frac{1}{2}-\frac{3}{7}$ **b** $2\frac{2}{3}\times1\frac{3}{4}$. (4024 paper 01 Q1 June 2008)

9 Evaluate
a $25-18.3$ **b** 1.7×0.03. (4024 paper 01 Q2 June 2008)

10 a Evaluate 0.5×0.007. **b** Evaluate $\frac{1}{1.25}$ as a decimal. (4024 paper 01 Q5 June 2009)

11 Arrange these values in order of size, starting with the smallest:

$\frac{9}{20}$ 0.39 46% $\frac{2}{5}$ (4024 paper 01 Q3 November 2009)

12 a Evaluate $3+25\div2$. **b** Express $17\frac{1}{2}\%$ as a decimal.
(4024 paper 01 Q1 June 2007)

13 Evaluate
a $\frac{1}{4}+\frac{1}{7}$, **b** $1\frac{7}{8}\div\frac{3}{16}$. (4021 paper 01 Q2 June 2007)

14 It is given that $\frac{2}{3}, \frac{8}{d}$ and $\frac{n}{39}$ are equivalent fractions.

Find the value of d and the value of n. (4024 paper 01 Q3 June 2007)

15 a Write 3% as a fraction. **b** Work out $90-16\div2$.
(4024 paper 11 Q3 November 2014)

16 Evaluate

a $10-7.56$, **b** 0.105×0.2. (4024 paper 01 Q1 November 2005)

17 a Evaluate $3+5(3-1.4)$. **b** Evaluate 0.2×0.07.
(4024 paper 11 Q1 November 2011)

18 Evaluate
a $3+2(4-5)$ **b** $1\frac{1}{3}\div2\frac{1}{2}$. (4024 paper 01 Q1 November 2006)

19 a Write the following in order of size, starting with the smallest:
$\frac{66}{100}$ $0.\dot{6}$ 0.67 $\frac{666}{1000}$

b The distance of Saturn from the Sun is 1507 million kilometres.
Express 1507 million in standard form. (4024 paper 01 Q5 June 2007)

Beginning Algebra 3

Learning Objectives

Syllabus sections 4, 17, 18, 19

In this chapter you will begin your study of:

- the language of algebra
- addition, subtraction, multiplication and division
- directed numbers
- indices, brackets and common factors in algebra.

3.1 Introduction

Algebra is a tool for doing arithmetical calculations when some of the numbers needed are unknown. The rules of algebra help us either to calculate the values of these numbers, or to find formulae which can be used to make calculations later when some of the numbers are known. The formulae may link two or more unknown numbers. If these unknown numbers can take different values they are called variables.

Learning to use algebra is like learning a language. We need clear rules for the language so that we can all understand each other. You have met rules like these before, in Chapter 1, when you learned that the same order of working in arithmetic is needed if we are all to get the same answer.

3.2 Essential Skills NO CALCULATOR IN THIS EXERCISE

1 Calculate:
  a $2 \times 6 + 3 \times 5$
  b $3 \times (6 - 4)$
  c $1 + 2 \times 3 - 4 \div 2 + 5 \times (6 - 3)$

2 **i** What is the sum of 5 and 6?
  ii What is the product of 5 and 6?

3 **i** What is the HCF of 20, 45 and 15?
  ii Rewrite 20, 45 and 15 as products of this factor and one other in each case

Key term

Variables are usually letters which represent numbers or amounts that can change or be given different values.

3.3 Using Letters and Numbers

Letters as Variables

Suppose you are going to buy 3 apples and 5 oranges. If you know the price of both fruits, you can work out what the total cost will be. Suppose the apples cost 10 cents each and the oranges cost 12 cents each, then the total cost, in cents, will be:

$$3 \times 10 + 5 \times 12$$

Using the correct order of working for arithmetic we can finish this:

$$\text{Total cost} = 3 \times 10 + 5 \times 12$$
$$= 30 + 60 = 90 \text{ cents}$$

But suppose we do not know the cost of the apples?

We can still do some of the work like this:

$$\text{Total cost (in cents)} = 3 \times \textit{cost of an apple} + 5 \times 12$$
$$= 3 \times \textit{cost of an apple} + 60$$

This would take too much time to keep writing out.

If we use a to mean the number of cents that an apple costs, then the sum becomes

$$\text{Total cost (in cents)} = 3 \times a + 5 \times 12$$
$$= 3 \times a + 60$$

We can make this look neater by using one of the rules of algebra, that $3 \times a$ can be shortened to $3a$.

Our final statement is:

$$\text{Total cost (in cents)} = 3a + 60$$

which can also be written

$$\text{Total cost} = (3a + 60) \text{ cents}$$

where a represents the *number* of cents you have to pay for an apple.

Later, when we know the cost of an apple we can finish the sum.

The number of cents is a *variable*. It could be 10 cents today and 12 cents tomorrow.

The total number of cents, or cost, is also a variable, but depends on a.

It is very important that you understand that the letter a stands for a number, not an apple or a number of apples. The statement should really be written:

$$\text{Total cost (in cents)} = 3 \times a \text{ cents} + 60 \text{ cents}$$

but it is usually sufficient to use the word cents once only, *or* to explain that the whole calculation is in cents.

Algebraic Shorthand

To get started with algebra we must start to learn a few rules. We will often use x and y as our unknown quantities, but remember that we can use any letter. When two or more letters are different, we know that they are being used for *different* numbers.

REMEMBER:

- $x+x+x+x+x+x+x+x+x=9\times x$ and $y+y+y=3\times y$.
- $9\times x$ can be shortened to $9x$, so $9\times x=9x$ and $3\times y=3y$.

REMEMBER ALSO:

- $x\times x=x^2$ and $y\times y\times y=y^3$.
- $1x$ can be written as just x, so $1x=x$ and $1y=y$.

These also agree with our work with numbers:

$2\times 2\times 2=2^3$ and $1\times 2=2$

Key terms

Simplify means write in its simplest form.

Solve usually means find a numerical solution to a problem or equation.

Calculate means find a numerical answer.

We start with some shorthand.

Had you noticed that multiplication is a shorter form of addition?

There are two ways to work out $7+7+7+7+7+7+7+7+7$.

You can either go through and add sevens as you go along, or you can see that there are 9 sevens, and quickly get the answer:

$$9\times 7=63$$

You can do this with any number, not just 7, so we could call the number x.

$9\times x$ can be shortened to $9x$ without any confusion, but 9×7 cannot be shortened to 97, which is completely different.

How Algebra is Similar to Arithmetic (and How it is Different)

When we are using letters and numbers, **simplify** is an instruction to write an answer in its simplest form. The answer will usually still contain letters.

Solve usually means to find a numerical answer to an equation.

When we are using numbers only, **calculate** is an instruction to find the solution to a numerical problem. The answer will be a number, or numbers.

We can *simplify* $y\times y\times y$ to y^3, but $2\times 2\times 2=2^3$ $(=8)$ can be *calculated* to give the numerical answer 8.

Much of your algebra will involve simplifying or, later, writing things in another form.

Work through the next example, paying particular attention to the numerical questions which show the similarities between algebra and arithmetic, and the difference between simplifying and calculating.

Example 1

Simplify the following:	*Calculate* the following:
a $x+x+x+x+x$	**b** $2+2+2+2+2$
c $y\times y$	**d** 8×8
e $4x+2x$	**f** $4\times 7+2\times 7$
g $3x+x$	**h** $3\times 5+5$
i $3y-y$	**j** $3\times 12-12$
k $x+x+y+y+y$	**l** $115+115+108+108+108$
m $9x-8x$	**n** $9\times 157-8\times 157$

Answer 1

a $x+x+x+x+x=5x$	**b** $2+2+2+2+2=5\times 2=10$
c $y\times y=y^2$	**d** $8\times 8=8^2=64$
e $4x+2x=6x$	**f** $4\times 7+2\times 7=6\times 7=42$
g $3x+x=4x$	**h** $3\times 5+5=4\times 5=20$
i $3y-y=2y$	**j** $3\times 12-12=2\times 12=24$

k $x+x+y+y+y=2x+3y$

l $115+115+108+108+108$
$=2\times115+3\times108$
$=230+324$
$=554$

m $9x-8x=1x=x$

n $9\times157-8\times157=1\times157=157$

REMEMBER:

- **Simplify** means to write in the simplest form.
- **Solve** usually means to find a numerical answer to an equation.
- **Calculate** means to find a numerical answer.

If you feel tempted to go further with Example 1 part **k** and attempt some sort of addition of the xs and ys, try it with the numbers as well and see if it works.

For example, *if* you think that $2x+3y$ could be shortened to $5xy$ (which of course you should not!), use numbers to check it.

$2x+3y$: $2\times115+3\times108$
$=230+324$
$=554$

but $5xy$: $5\times115\times108$
$=62100$

which is clearly not the same as 554!

We can only arrive at a final answer when we know what numbers will replace x and y. Until then the question has to ask you to *simplify*, rather than *solve*.

Exercise 3.1

NO CALCULATOR IN THIS EXERCISE

Copy and complete Table 3.1 by using algebraic shorthand to simplify, and the rules of arithmetic (BoDMAS) to calculate.

		Simplify	Answer		Calculate	Answer
a	**i**	$x+x+x$		**ii**	$30+30+30$	
b	**i**	$5y-4y$		**ii**	$5\times154-4\times154$	
c	**i**	$z\times z\times z$		**ii**	$3\times3\times3$	
d	**i**	$x+x+x-y$		**ii**	$6+6+6-10$	
e	**i**	$x+x+y+y$		**ii**	$7+7+4+4$	
f	**i**	$y-y$		**ii**	$2-2$	
g	**i**	$x\times x+y\times y$		**ii**	$3\times3+4\times4$	
h	**i**	$5x+3x-2y$		**ii**	$5\times50+3\times50-2\times4$	

Table 3.1

3.4 The Language of Algebra

Expressions, equations and terms

There are some other words that have a special meaning in algebra, and you must understand them as well. First of all, try to understand the difference between an *expression* and an *equation*. Have a look at this piece of algebra:

$$3x+5y-10z+6$$

Key terms

In algebra, **terms** are numbers and letters that are added or subtracted. For example, in $3x + 5y$, $3x$ and $5y$ are terms. $3x$ is a **term in x** and $5y$ is a **term in y**.

Expressions are groups of terms to be added or subtracted. They do not have an equals sign. They cannot be solved, but may be simplified.

The **coefficient** of a term is the number in front of it, for example the coefficient of $3x$ is 3.

Like terms have the same letters, for example $4z$ and $10z$.

An **equation** has an equals sign and can often be solved.

This is an **algebraic expression.** It is not an equation since it stands alone without an equals sign. It is made up of **terms** which are to be added or subtracted. The terms are $3x$, $5y$, $10z$ and 6.

$3x$ is a 'term in x', $5y$ is a 'term in y', $10z$ is a 'term in z', and 6 is a constant or a number term.

The 6 is a constant term because it is always 6, but $3x$ is not constant because it depends on what x stands for.

Each number in front of a term is the **coefficient** of that term.

Now look at the following expression:

$$2x + 7y - 3y + 4x$$

This is an expression that can be simplified. It has **like terms.** It has two terms in x and two terms in y. We can write:

$$\begin{aligned} &2x + 7y - 3y + 4x \\ &= 2x + 4x + 7y - 3y \\ &= 6x + 4y \end{aligned}$$

This is called **collecting like terms**.

Each of the two equals signs shows that the next line is equivalent to the one before, but has been written in another way. They do *not* convert the expression into an equation.

But if we are given a bit more information, for example, that our expression is actually equal to something else, we have an equation.

For example, $6x + 4y = 34$ is an **equation**.

An expression is like a phrase in English, and an equation is more like a sentence. For example, 'hot and stormy weather' is a phrase in English. It means more when it becomes a sentence such as: 'Today we are having hot and stormy weather' and we have the extra bit of information that it is today that we are talking about.

An equation may be **solved** by finding replacements for the variables which make it a true statement. For example, we can solve $10z - 3 = 17$.

This is an equation which becomes true when z is replaced by 2.

$$10 \times 2 - 3 = 17$$

So the solution to the equation is $z = 2$, and in this case it is the only solution. The equation $6x + 4y = 34$ becomes true when we replace the x by 3 and the y by 4, because

$$6 \times 3 + 4 \times 4 = 18 + 16 = 34$$

So $x = 3$ and $y = 4$ is a solution to this equation.

In this case this is not the only possible solution.

For example, $x = 2.5$ and $y = 4.75$ is also a solution. Check it for yourself!

So far we have mainly used letters to represent unknowns or numbers. But remember the example of buying apples and oranges?

We wrote: Total cost (in cents) $= 3a + 60$.

Here a represents the variable cost of one apple, in cents.

Variables can be represented by words, letters or symbols.

NOTE:

NEVER try to turn an expression into an equation, for example by making it equal to zero, unless the question asks you to. This is a common mistake made by students.

Remember that you may be able to simplify an expression, but not solve it. You may be able to simplify *and* solve an equation.

For example,

$$3 \times what = 21$$
$$3x = 21$$
$$3 \times ? = 21$$
$$3 \times \square = 21$$

In each case the unknown can be replaced by 7 to make the equation true. For simplicity it is usual to use letters.

Example 2

a $3x + 4y + y = 3x + 5y$ $7x + 10y = 37$ $3a - 4b$

From the above, select:

i a term in x
ii a pair of like terms
iii an equation
iv an expression which is then simplified
v another expression
vi a constant term
vii the coefficient of the term in b.

b Can you find replacements for x and y that would make $7x + 10y = 37$ true?

c I give a shopkeeper 10 cents. He gives me 4 mangoes and 4 cents change.
Write an equation to show this and so find the price of one mango.

d **i** Use the letters given to write an equation to represent the following statement:
'I buy 2 bags of crisps and 3 chocolate bars. I spend 12 cents altogether.'
Use x = the cost, in cents, of a bag of crisps, and y = the cost, in cents, of a chocolate bar.
ii Find one pair of possible replacements for x and y which would make your equation true.

Answer 2

a **i** $3x$ or $7x$ are both terms in x.
ii $4y$ and y are like terms.
iii $7x + 10y = 37$ is an equation.
iv $3x + 4y + y$ is an expression which is simplified to $3x + 5y$.
v $3a - 4b$ is another expression.
vi 37 is a constant term.
vii 4 is the coefficient of the term in b.

NOTE:
Remember that $3x + 4y + y = 3x + 5y$ is not an equation.

b $7x + 10y = 37$
By trying a few numbers we find that $x = 1$ and $y = 3$ would make this equation true.
$7 \times 1 + 10 \times 3 = 7 + 30 = 37$
If you use rational numbers there are an infinite number of solutions.
For example, $x = 2$ and $y = 2.3$. Can you find some more?

c If m = the cost of one mango, in cents,
$10 = 4m + 4$ (10 cents = $4 \times m$ cents + 4 cents change)
so $4m = 6$
so $m = 1.5$
Hence, mangoes cost 1.5 cents each.

d **i** $2x + 3y = 12$
ii $x = 3$ and $y = 2$ is one possible pair of values that would make this true.

NOTE:
We can only use counting numbers here. Why?

Formulae and Substitution

Key term

A **formula** (plural **formulae**) is used to calculate quantities, for example

$\text{speed} = \frac{\text{distance}}{\text{time}}$.

Formulae are equations that are used fairly frequently to calculate quantities and are arranged so that the required quantity is the **subject** of the formula. This makes them convenient to use. For example, in our original small problem of the cost of apples and oranges, we ended up with the formula

$$\text{Total cost (in cents)} = 3a + 60$$

where a represents the cost of each apple in cents.

This can be called a formula because it is arranged so that the quantity we want to find (total cost) is on its own on the left and so is the subject of the statement.

When we know the cost of an apple we will be able to substitute this in to replace a, and calculate the total cost. Suppose b is the cost of one orange, in cents, and T is the total cost, in cents; then our formula could become more useful:

$$T = 3a + 5b$$

T, a and b are the unknowns or variables because the prices may vary from day to day.

When we know the cost of an apple and an orange on the day we can **substitute** these numbers for a and b and work out the total cost on the day.

Key term

Substitution is replacing an unknown variable by a number so that a formula or expression may be evaluated.

Substitution is replacing unknowns or variables by numbers, usually in formulae, so that answers may be calculated. Calculating the answer when variables in an expression are substituted by numbers is often called **evaluating** the expression.

Example 3

a I think of a number (n), multiply it by 4, add 6, then take away the number I first thought of.
Write a formula for the answer (A) in terms of n.

b Use the formula to find A when:
i $n = 3$ **ii** $n = 100$ **iii** $n = 11$

Answer 3

a $A = n \times 4 + 6 - n$
$A = 4n + 6 - n$
$A = 3n + 6$

b
i when $n = 3$
$A = 3 \times 3 + 6 = 9 + 6$
$A = 15$

ii when $n = 100$
$A = 3 \times 100 + 6$
$A = 306$

iii when $n = 11$
$A = 3 \times 11 + 6$
$A = 39$

Exercise 3.2

NO CALCULATOR IN THIS EXERCISE

1 Maria is m years old. Her father, Bakari, is n years *older* than Maria.
Write an expression for the sum of their ages.

2 A piece of wood is 6.5 metres long. Brian saws off and uses m metres.
Write an expression for the length of wood which is left.

3 Amir starts his journey to school by walking for 10 minutes, and then takes a bus. The time (t minutes) the bus takes to get to the school depends on the traffic.

a Write a formula for the total journey time (T minutes) in terms of t.

b Find T when $t = 15$.

4 Substitute $y = 3$ and $z = 5$ into each of the formulae below to find x.

a $x = 2y + 3z$ **b** $x = yz + 2$ **c** $x = 4yz - 3z + 2y$

5 Figure 3.1 shows a triangle with two sides of length a cm, and one side of length 3 cm.

a Write a formula for the total length (L cm) round the outside of the triangle.

b Use the formula to find L when $a = 10$.

c Why can a not be

i 1.5? **ii** 1?

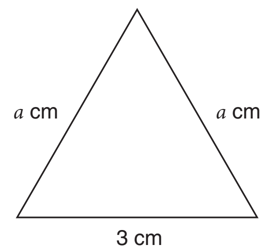


Figure 3.1

6 A recipe requires 5 eggs, 0.5 kilograms of butter and 0.5 kilograms of tomatoes.
Eggs cost e cents per 10, butter costs b cents per half kilogram and tomatoes cost t cents per kilogram.

a Write a formula for the total cost (C cents) of the recipe.

b Calculate C when e = Rs. 22, b = Rs. 58 and t = Rs. 12.

7 Evaluate the following expressions when $x = 2$ and $y = 3$.

a xy **b** $y - x$

c $y^2 - x^2$ **d** $3x + 9y$

3.5 Addition and Subtraction of Terms in Algebra

As we have already seen, addition and subtraction of terms in algebra is also called *collecting like terms*.

We could illustrate the process by thinking of a zoo which keeps antelopes and bears.

Antelopes and bears themselves are not variables or unknown quantities, so we will not replace them with letters. This example is just to help you understand how terms can be moved within an expression as long as the sign in front of the term stays with that term.

Think: 'the sign belongs to the term that follows it'.

The zoo starts with 10 antelopes and 5 bears, and then they trade 4 of their antelopes for 2 bears with another zoo. Later on they give away 2 more antelopes. How many antelopes and bears do they now have? The situation could be written like this:

10 antelopes + 5 bears − 4 antelopes + 2 bears − 2 antelopes

which can be rearranged to:

10 antelopes − 4 antelopes − 2 antelopes + 5 bears + 2 bears

This gives us:

4 antelopes + 7 bears

So you can see that we can rearrange the sum as long as we keep the sign with the animal that follows it.

REMEMBER:

- The sign belongs to the term that follows it.
- Terms can be written in any order in the expression as long as they keep their signs.
- Only like terms can be added or subtracted.